Dynamics and Control of Buckling Type Devices using SMA wire Integrated Beam

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Overview

- Modeling of Shape Memory Alloy (SMA) behaviour, review of constitutive model, transformation phase space
- Scope and importance of present research
- Evolution kinetics and scaling parameters.
- Updated Lagrangian finite element formulation
- Analysis of buckling shape control of a thin beam integrated with SMA wire
- Results and Concluding Remarks

Pseudo-elastic Response of SMA

- SMA undergoes large thermo-mechanical phase transformation (Martensite to Austenite and vise versa).

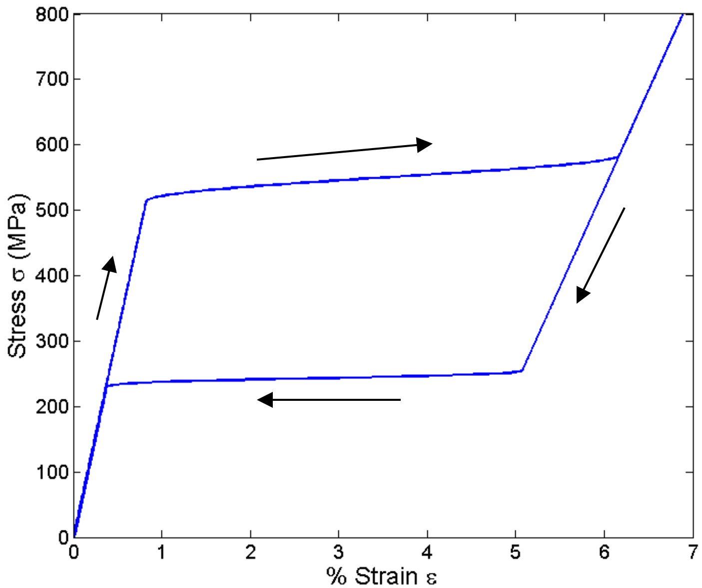
- Exhibits ‘pseudo-elasticity’ and ‘shape memory’ effects.

- Pseudo-elastic effect – large inelastic strain due to mechanical loading fully recovered in a hysteresis loop upon unloading.

- Temperature is maintained constant and above Austenite finish.

$$(T > A_f\ , \qquad \dot{T} = 0)$$

$$\text{(Austenite)} \xleftrightarrow{\sigma} \text{(‘Twinned’ Martensite)}$$



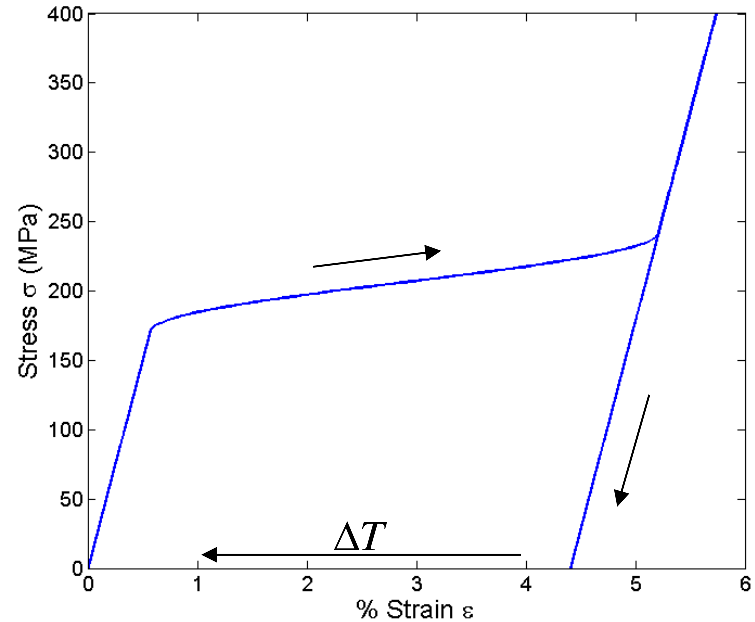
Typical SMA wire pseudo-elastic response

Shape Memory Effect in SMA

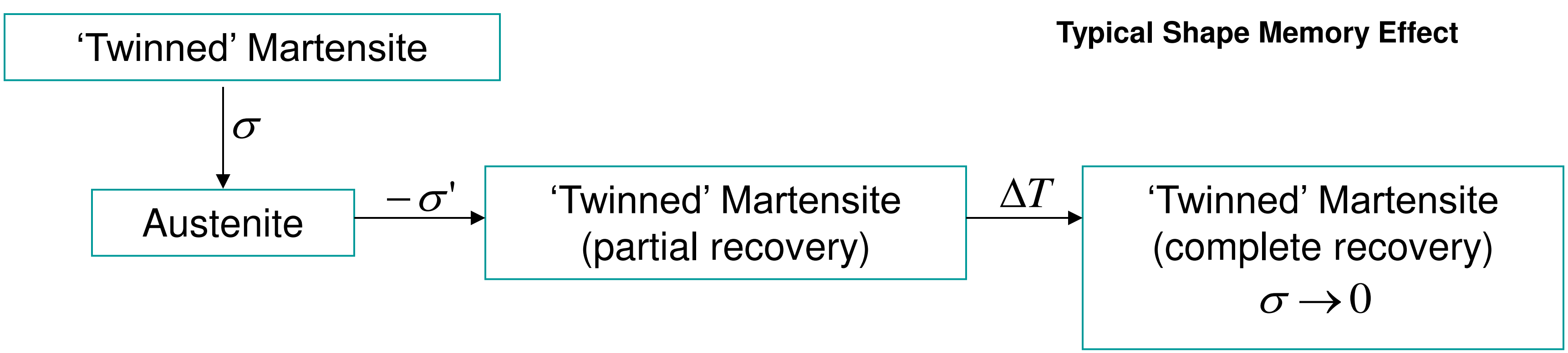
- Partial recovery of strain upon mechanical unloading (T=constant)

- Full recovery after heating above austenite finish temperature ($T>A_f$)

- Heating to a stress-free condition gives back ‘Twinned’ martensite



Typical Shape Memory Effect



Constitutive Model of SMA Wire

- One-dimensional phenomenological model by Brinson (1993).
- Modification - Differential form of the constitutive equation after modifying the transformation tensor for thermodynamic reciprocity (Khandelwal and Buravalla 2006)

$$d\sigma = D(\xi)d\epsilon - \epsilon_l D(\xi)d\xi_S + (\epsilon - \epsilon_l \xi_S)(D_m - D_a)d\xi + \Theta dT$$

- Integrated form of the constitutive

$$\sigma - \sigma_0 = D(\xi)\epsilon - D(\xi_0)\epsilon_0 - \epsilon_l D(\xi_0)\xi_{S0} + \Theta(T - T_0)$$

$$\xi = \xi_S + \xi_T$$

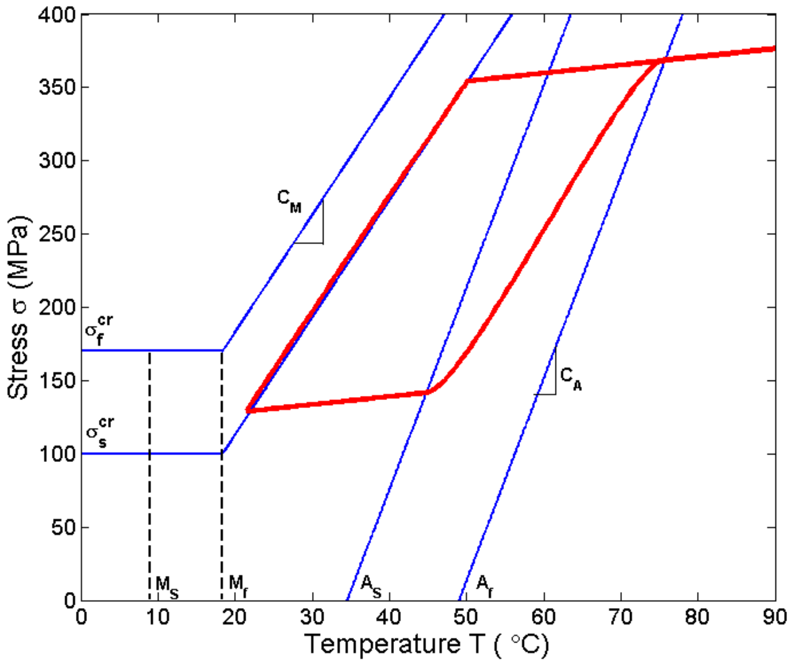
- Incremental form of the constitutive equation

$$\begin{aligned}\Delta\sigma = &(D_m - D_a)\epsilon_0\Delta\xi + (D_m - D_a)\Delta\xi\Delta\epsilon + D_0\Delta\epsilon + \Theta\Delta T \\ &-\epsilon_l\{(D_m - D_a)\xi_{S0}\Delta\xi + D_0\Delta\xi_S + (D_m - D_a)\Delta\xi\Delta\xi_S\}\end{aligned}$$

Transformation Phase Space

- Phase transformation is determined by competition between instantaneous stress, temperature and critical (transformation) stress and temperature

- Critical stress and temp's represent the phase boundaries in the $(\sigma - T)$ space and divides it into various zones.



Response under constant elastic strain

$M_S,\ M_f$ - Martensite start and finish temperatures.

$A_S,\ A_f$ - Austenite start and finish temperatures.

$\sigma_s^{cr},\ \sigma_f^{cr}$ - Critical transformation stresses start and finish.

$C_M,\ C_A$ - Slope of the phase boundaries

SMA Based Devices for Buckling Type Control

- SMA's ability to recover from large inelastic deformations
  - A popular choice for applications with large deformations and slow dynamics

- SMAs can be employed for active stabilization of structures undergoing large deformations

- Martensite evolution in the SAM wires – competition between thermo-mechanical force and interface driving force

- Underlying transformation path is non smooth.

- Continuum theory – point-wise description of martensitic states (an approxation of true microstructural phases in a spatially averaged sense)

SMA Based Devices for Buckling Type Control

- Phase inhomogeneity is Inevitable due to responses in the fast time scales. Therefore, such physical details key towards understanding SMA defects and successfully design SMA integrated systems

- Evolution kinetics relates rate of formation of martensite to various driving forces which in turn depend upon dissipation of free energy and non-local effects

- Since martensite evolution depends upon interplay between thermo-mechanical force and interface driving force, it is essential to study these processes (wire drawing, cold-works, annealing, cyclic stabilization) in context of each individual type of design applications.

Background Literature

Constitutive Model

• Brinson (1993), One-dimensional phenomenological model with non-constant material functions.

• Buravalla and Khandelwal (2006), Modified transformation tensor to satisfy consistency of differential and integrated form of Brinson's 1D phenomenological model.

Evolution Kinetics

• Brinson (1993), phase diagram based approach
– transformation between Ms and As not considered, no rate effects.

• Roy Mahapatra and Melnik (2006), Free energy based variational framework including kinetics.

Background Literature

Finite Element Formulation

- Brinson and Lammering (1993), Nonlinear finite element formulation based on Green Lagrange strain and consistent linearization.

- Serry, Raboud and Moussa (2003), Sequential solution of nonlinear finite element formulations (structural and transient electro-thermal). Relies on consistent linearization of Green Lagrange strain.

- Gao, Qiao and Brinson (2007), Finite element implementation in ABAQUS based on phenomenolgical phase diagram.

Development of Analysis and Design Tools

- Introduction of kinetics scaling parameters systematically to investigate the effect of microstructural variation on thermo-mechanical forces and interface driving forces forces and consequently on martensite evolution.

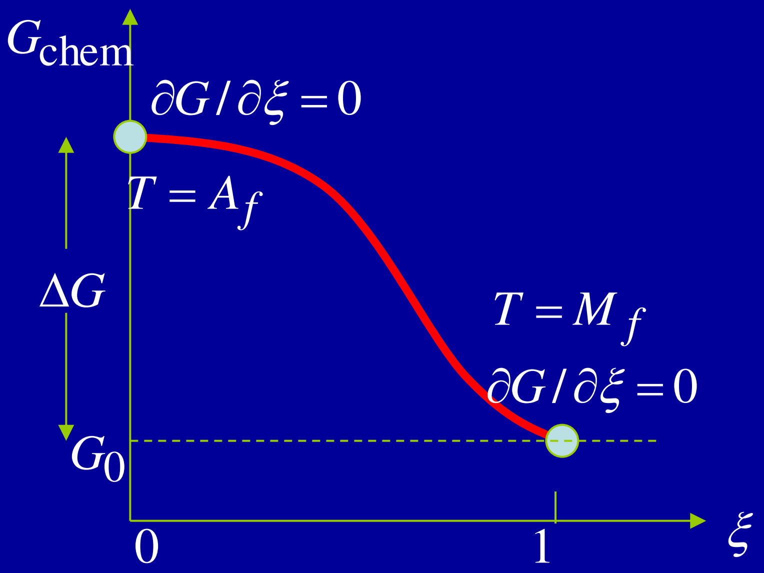
- Development of a finite element model of SMA wire, based on incremental form of the one-dimensional constitutive equation and using updated lagrangian, to solve the boundary value problem by accommodating for phase inhomogeneity which is present in the SMA wire.

- Design and analysis of shape control methodology of a SMA wire integrated beam.

Identification of Various Scales in Microstructure Evolution in Phenomenological Model

$$\dot{\xi} = A\frac{\partial G}{\partial \xi} + B\nabla^2\xi + \cancel{\overline{\Theta}(T)}$$

$$G = -\frac{1}{2}\sigma^T\varepsilon_{\mathrm{el}} + G_{\mathrm{chem}}(T,\xi), \qquad G_{\mathrm{chem}} = G_{\mathrm{chem}}^A - G_{\mathrm{chem}}^M$$



Let us assume

$$G = (T - T_e)(a_0 + a_1\xi + a_2\xi^2 + a_3\xi^3)$$

By applying the transformation conditions for $\varepsilon_{\mathrm{el}} = 0$

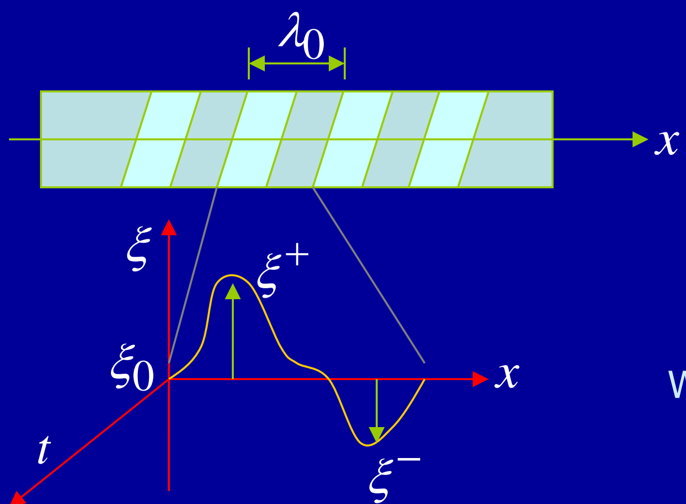
$$\dot{\xi} = A\overline{A}_1(T,\Delta G)\xi(1-\xi) + B\nabla^2\xi$$

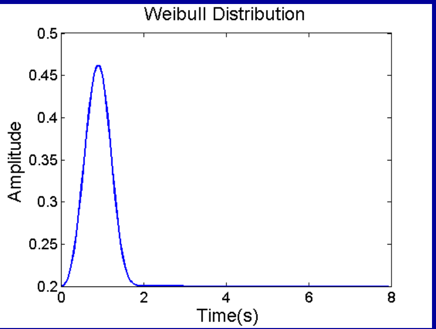
where

$$\overline{A}_1 = -6\left[G_0\frac{M_f - A_f}{(M_f - T_e)(A_f - T_e)} + \Delta G\frac{1}{(A_f - T_e)}\right]$$

Identification of Various Scales in Microstructure Evolution

Alternate variants and phase boundaries:





$$\xi(x,t) = \xi_0 + \Delta\xi(x,t)$$

$$0 \quad \le \quad \xi_0 + \Delta\xi(x,t) \quad \le \quad 1$$

$$\Rightarrow 0 \quad \le \quad \int_{-\infty}^{+\infty} (\xi_0 + \Delta\xi(x,t)) e^{-i\omega t} dt \quad \le \quad \int_{-\infty}^{+\infty} e^{-i\omega t} dt$$

Weibull distribution for nucleation:

$$\Delta\xi(x,t) = \hat{\xi}_0(x) W(t,k), \qquad W(t,k) = \frac{k}{\Delta\tau}\left(\frac{t}{\Delta\tau}\right)^{k-1} e^{-\left(\frac{t}{\Delta\tau}\right)^k}$$

k : Shape parameter

$\Delta\tau$: Time scale parameter

After Fourier transform from time to frequency domain,

$$\nabla^2\hat{\xi}(\omega) = \frac{\hat{\xi}(\omega)_{j-1} - 2\hat{\xi}(\omega)_j + \hat{\xi}(\omega)_{j+1}}{\lambda_0^2/4} = \frac{2\xi_0' - 2\hat{\xi}(\omega)_j}{\lambda_0^2/4}$$

Main Results

$$\dot{\xi} = A\frac{\partial G}{\partial \xi} + B\nabla^2 \xi + \overline{\Theta}(T)$$

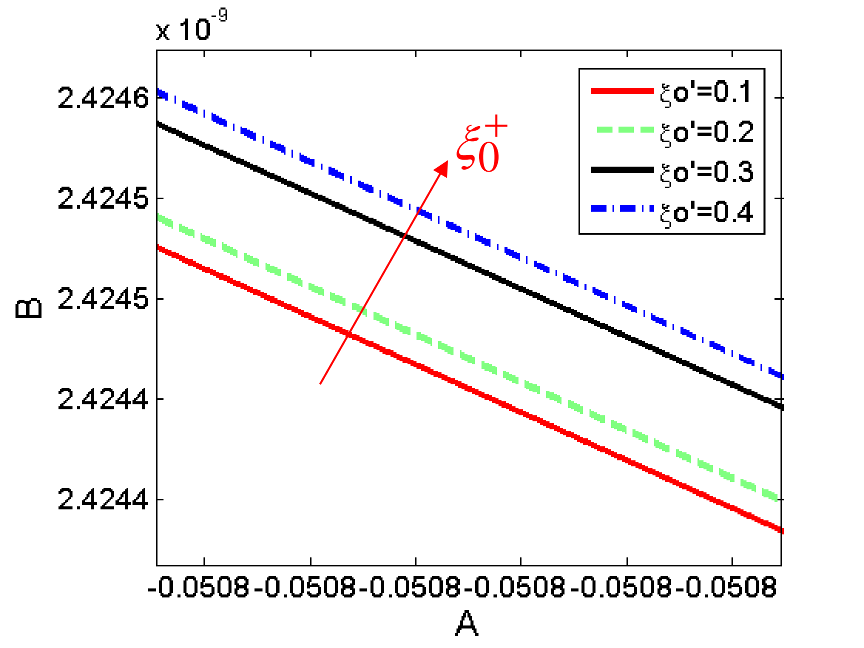
Under zero elastic strain ($\varepsilon_{\mathrm{el}} = 0$)

$$B = \frac{\lambda_0^2 \omega}{8\xi_0^{'} \operatorname{Im}[\hat{\xi}(\omega)_j]}$$

$$A = \frac{\omega(1-\xi_0^{'})\operatorname{Re}[\hat{\xi}(\omega)_j]}{(T-T_e)\overline{A}_1(1-2\xi_0^{'})\xi_0^{'}\operatorname{Im}[\hat{\xi}(\omega)_j]},$$

$$\overline{A}_1 = -6\left[G_0\frac{M_f - A_f}{(M_f - T_e)(A_f - T_e)} + \Delta G\frac{1}{(A_f - T_e)}\right]$$

$$1 + \frac{\xi_0 |\hat{\xi}(\omega)|^2}{\operatorname{Re}[\hat{\xi}(\omega)_j]\cos(\omega t) + \operatorname{Im}[\hat{\xi}(\omega)_j]\sin(\omega t)} \le 0$$



Main Results

$$\dot{\xi} = A\frac{\partial G}{\partial \xi} + B\nabla^2\xi + \overline{\Theta}(T)$$

Under finite elastic strain ($\varepsilon_{\text{el}} \neq 0$)

$$G = -\frac{1}{2}\sigma^T \varepsilon_{\text{el}} + G_{\text{chem}}(T,\xi)$$

$$B = \frac{\lambda_0^2 \omega}{8\xi_0' \operatorname{Im}[\hat{\xi}(\omega)_j]}$$

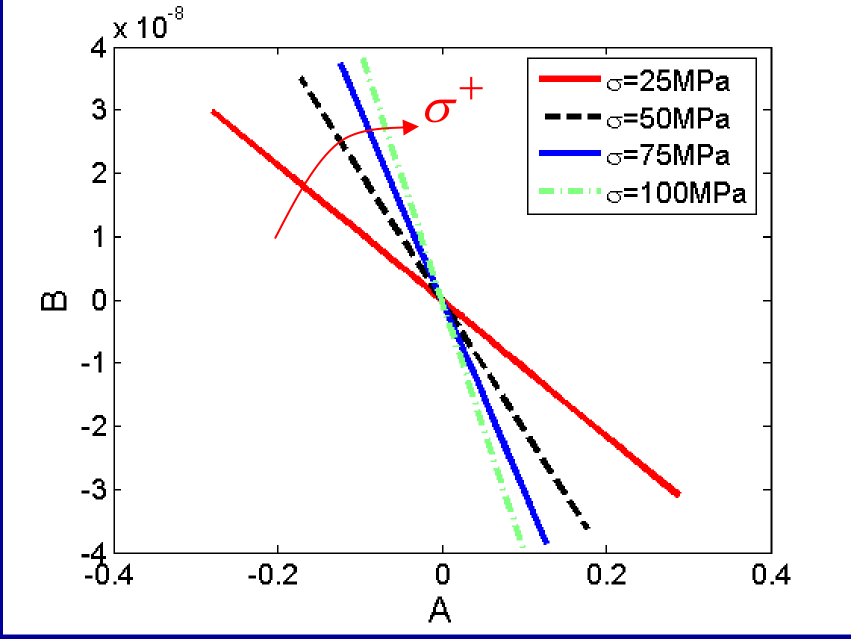
$$\sigma - \sigma_0 = D(\xi)\varepsilon_{\text{el}} - D(\xi_0)\varepsilon_0 + \Omega(\xi)\xi_S - \Omega(\xi_0)\xi_{S0} + \Theta(\xi)(T - T_0)$$

$$A = \frac{\omega(1-\xi_0')\operatorname{Re}[\hat{\xi}(\omega)_j]}{[(D_m - D_a)\bar{\varepsilon} + (T - T_e)\overline{A}_1(1 - 2\xi_0')]\xi_0' \operatorname{Im}[\hat{\xi}(\omega)_j]},$$

$$\overline{A}_1 = -6\left[G_0 \frac{M_f - A_f}{(M_f - T_e)(A_f - T_e)} + \Delta G \frac{1}{(A_f - T_e)}\right]$$

$$1 + \frac{\xi_0 |\hat{\xi}(\omega)|^2}{\operatorname{Re}[\hat{\xi}(\omega)_j]\cos(\omega t) + \operatorname{Im}[\hat{\xi}(\omega)_j]\sin(\omega t)} \le 0$$

$$\bar{\varepsilon} = \frac{1}{D_m}[\Delta\sigma + D_a\varepsilon_0 + \varepsilon_L D_m - \Theta_m(T - T_0)]$$



Finite Element Formulation

- Based on the incremental form of constitutive equation,

$$\Delta\sigma = f(\Delta\xi, \Delta\epsilon, \Delta\xi_S, \Delta T, \epsilon_0, \xi_0, \xi_{S0})$$

- Constructing weighted residual,

$$\int_\Omega \delta u \left[\frac{\partial\Delta\sigma}{\partial\Delta\xi}\frac{d\Delta\xi}{dz} + \frac{\partial\Delta\sigma}{\partial\Delta\epsilon}\frac{d\Delta\epsilon}{dz} + \frac{\partial\Delta\sigma}{\partial\Delta\xi_S}\frac{d\Delta\xi_S}{dz} + \frac{\partial\Delta\sigma}{\partial\Delta T}\frac{d\Delta T}{dz} + \frac{\partial\Delta\sigma}{\partial\epsilon_0}\frac{d\epsilon_0}{dz} + \frac{\partial\Delta\sigma}{\partial\xi_0}\frac{d\xi_0}{dz} + \frac{\partial\Delta\sigma}{\partial\xi_{S0}}\frac{d\xi_{S0}}{dz} - \rho_d \ddot{u} \right] dz = 0$$

- Galerkin finite element approximation

- Displacement $u^e = \sum_{j=1}^{n} N_j U j$ and Temperature $\Delta T^e = \sum_{j=1}^{n} N_j \Phi_j$

- Finite element form

$$\left[K\right]\left\{U\right\} + \left[K_T\right]\left\{\Phi\right\} - \left[M\right]\left\{\ddot{U}\right\} = \left\{f(\epsilon_0,\ \xi_0,\ \xi_{S0}, \Delta\xi,\ \Delta\xi_S)\right\} + \left\{\Delta\sigma_{Applied}\right\}$$

Finite Element Formulation

$$\left[K\right]\left\{U\right\}+\left[K_T\right]\left\{\Phi\right\}-\left[M\right]\left\{\ddot{U}\right\}=\left\{f(\epsilon_0,\ \xi_0,\ \xi_{S0},\Delta\xi,\ \Delta\xi_S)\right\}+\left\{\Delta\sigma_{Applied}\right\}$$

$$K=\int_\Omega B^T\left[D_a+(D_m-D_a)\xi_0+(D_m-D_a)\Delta\xi\right]\frac{1}{J}Bd\eta$$

$$K_T=\int_\Omega B^T\Theta Nd\eta,\quad M=-\int_\Omega N^T\left\{\rho_{d0}+(\rho_{dm}-\rho_{da})\Delta\xi\right\}Nd\eta$$

$$f=-\int_\Omega B^T\left[(D_m-D_a)\epsilon_0\Delta\xi+\epsilon_l(D_m-D_a)\xi_{S0}\Delta\xi\right.$$
$$\left.+\epsilon_l(D_m-D_a)\Delta\xi_S\Delta\xi+\epsilon_l D_a\Delta\xi_S+\epsilon_l(D_m-D_a)\xi_0\Delta\xi_S\right]d\eta$$

K - Stiffness matrix, U – Displacement vector,

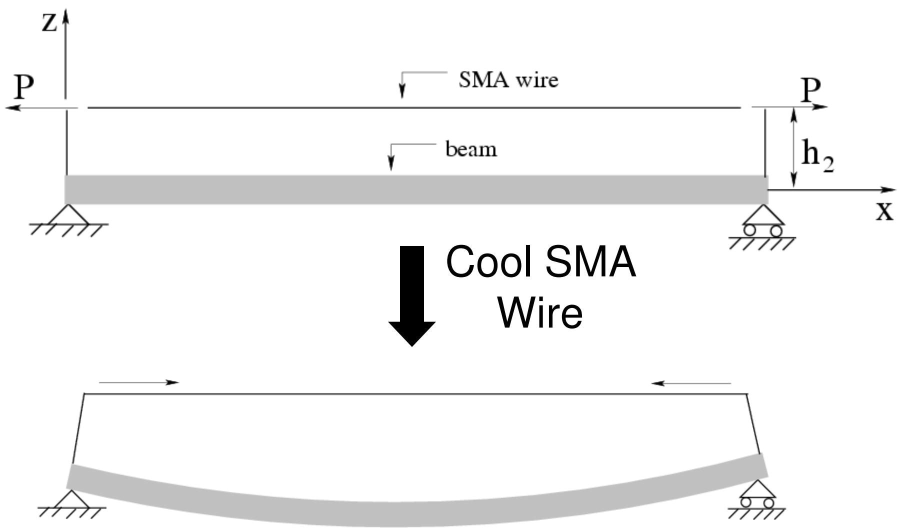
K_T - Stiffness due to temperature variation,

Φ - Vector containing temperature difference,

M - Mass matrix and f is the stress developed due to martensite evolution

Analysis of Buckling Shape Control of SMA wire integrated Beam

- SMA wire in Austenite state is pre-stressed and integrated with a thin beam. Span-wise deflection of the beam needs to be computed.



- Stress in the Longitudinal direction of the beam

$$\sigma_{xx} = E\left(\frac{\partial u_o}{\partial x} - z\frac{\partial^2 w}{\partial x^2}\right)$$

E – Modulus of elasticity

u_0 – Initial displacement

w – Transverse deflection

- For a given pre-stress in the wire, if it is possible to maintain a transverse deflection w, then post-buckling equilibrium equations and the boundary conditions must be satisfied.

Analysis of Buckling Shape Control of SMA wire integrated Beam

- Substituting σ_{xx} in the post buckled equilibrium

$$EI_0 \frac{\partial^2 u_0}{\partial x^2} - EI_1 \frac{\partial^3 (w - h_2)}{\partial x^3} = 0,$$
$$EI_1 \frac{\partial^3 u_0}{\partial x^3} - EI_2 \frac{\partial^4 (w - h_2)}{\partial x^4} = P(h_2 - w)$$

P – Applied load, h_2 – eccentricity of the wire
I_0, I_1, I_2 are the area moments of order 0,1 and 2.

- Assuming a harmonic solution $w - h_2 = w_0 e^{kx}$ and $u_0 = u_0 e^{kx}$, and substituting in the above,

$$EI_1 \bar{u}_0 k^3 - EI_2 w_0 k^4 + p w_0 = 0$$
$$EI_0 \bar{u}_0 k^2 - EI_1 w_0 k^3 = 0$$

- Solving these two equations, $k^4 \bar{I} E + P = 0$,

where $k = (\frac{P}{E\bar{I}})^{\frac{1}{4}},\ \bar{I} = \frac{I_1^2}{I_0} - I_2$

Analysis of Buckling Shape Control of SMA wire integrated Beam

- The general solution of $k^4\bar{I}E + P = 0$ given by

$$w = C_1 e^{kx} + C_2 e^{-kx} + C_3 x e^{kx} + C_4 x e^{-kx}$$

- The constants C_1, C_2, C_3 and C_4 are evaluated using boundary conditions for a simply supported beam, i.e.,

$$u_0 = 0, \;\; \frac{\partial w}{\partial x} = 0, \;\; \int \sigma_{xx} dA = 0, \;\; x = \frac{l}{2}$$

$$M_x = Ph_2, \;\; x = 0$$

$$M_x = Ph_2, \;\; x = l$$

$$w = 0, \;\; x = 0, l$$

$$C_1 = \frac{e^{-kl}(-e^{-kl} + e^{kl} + lke^{kl})h_2}{(e^{-kl} - e^{kl})^2} - \frac{1}{2}\frac{(-2e^{-kl} + 2e^{kl} + lke^{kl} + lke^{-kl})h_2}{(e^{-kl} - e^{kl})^2}$$
$$+ \frac{lPh_2}{k(e^{-kl} - e^{kl})^2(EI_1 - EI_2)} - \frac{1}{2}\frac{(e^{-kl} + e^{kl} lPh_2}{k(e^{-kl} - e^{kl})^2(EI_1 - EI_2)}$$

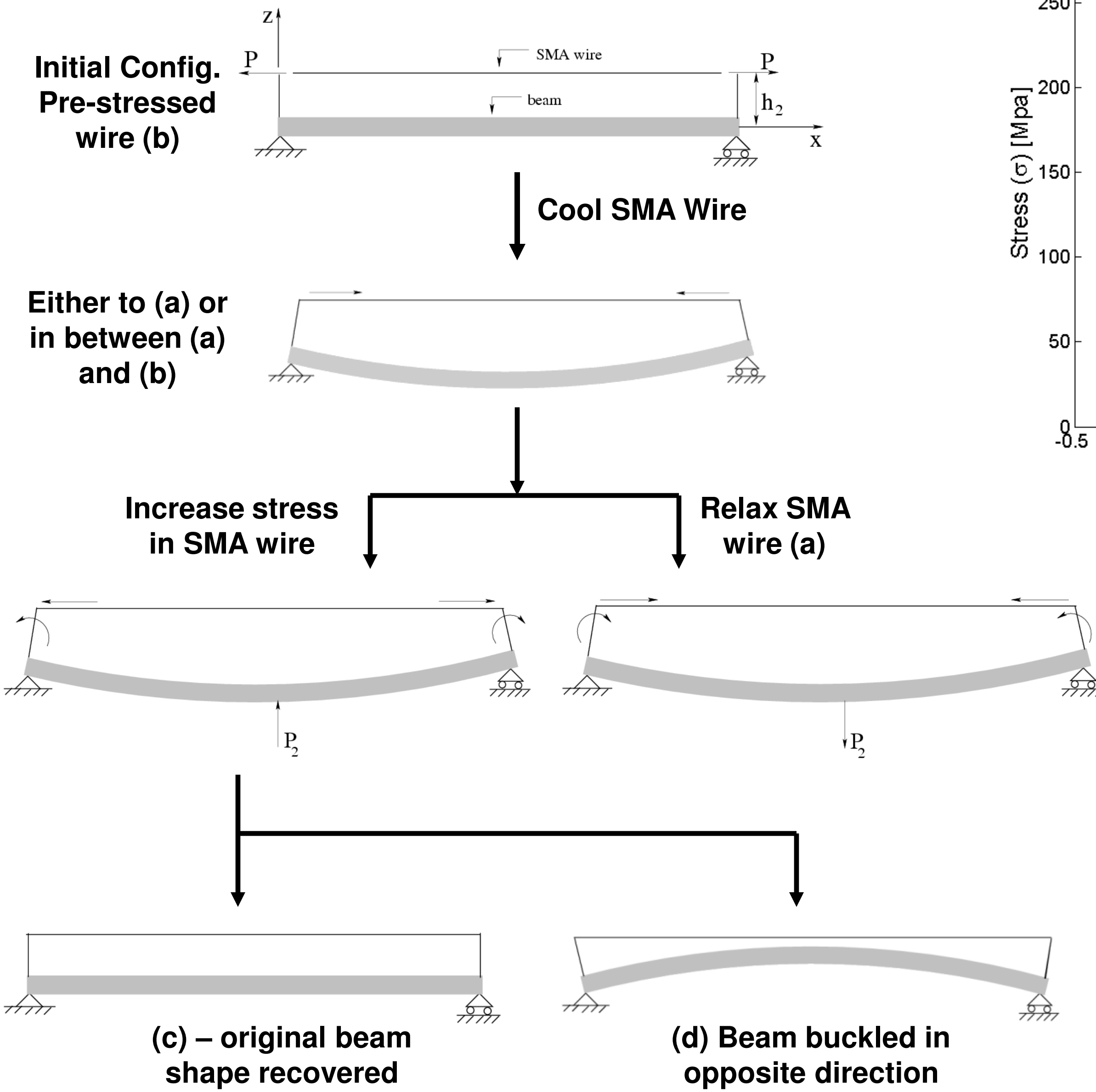
- Similarly, C_2, C_3 and C_4 are all functions of k.

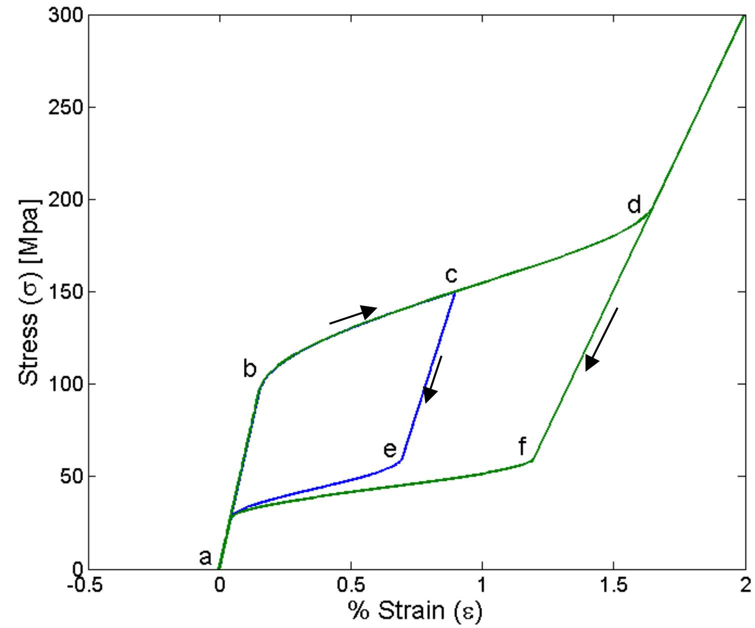
NiTi SMA wire material property

Moduli	Transformation Temperatures	Transformation Constants	Maximum Residual Strain
D_a = 63 GPA	M_f = 0	C_M = 3 MPa/K	ϵ_l =0.01
D_m = 30 GPa	M_s = 8	C_A = 3 MPa/K	
Θ = 0.55 MPa/K	A_s = 10	σ_s^{cr}= 30 MPa	
	A_f = 21	σ_f^{cr}= 130 MPa	

- Inertia effects in the SMA wire neglected
- Following simulations are for constant temperature application

Control Process





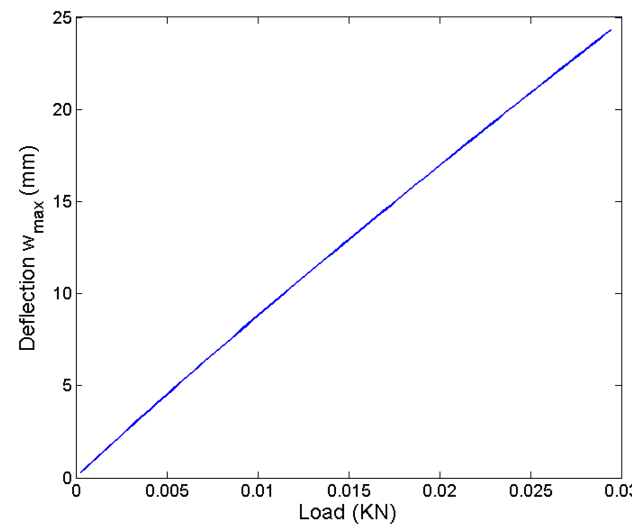
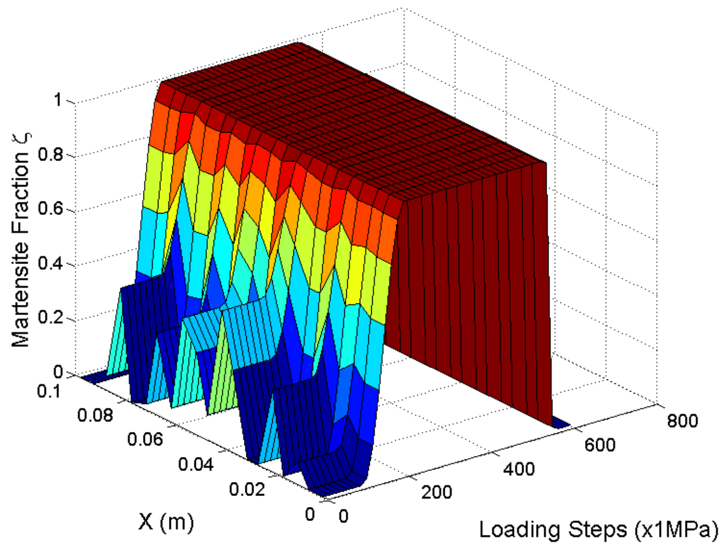
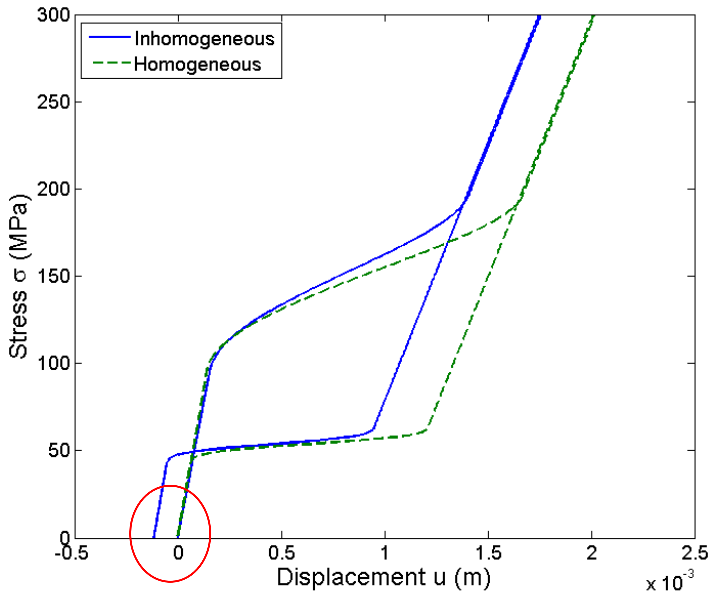


Figure: Maximum beam deflection

Finite Element Simulations



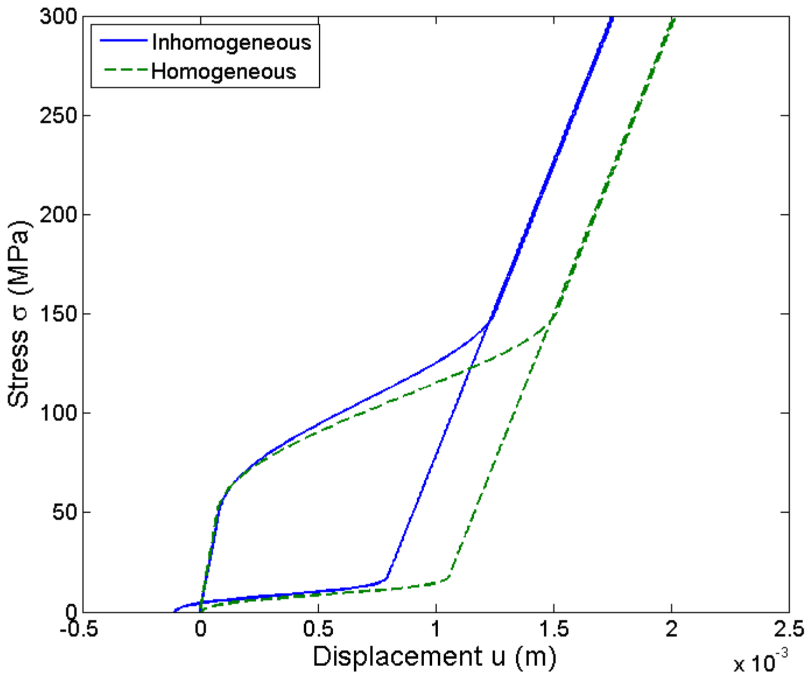
Martensite evolution across wire length



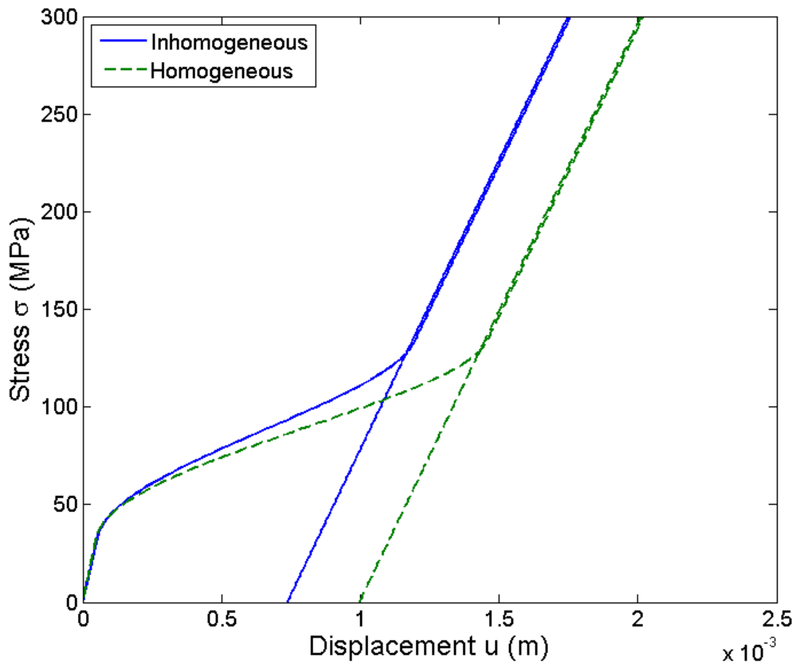
End displacement for T > A_f

- A random distribution of initial stress induced martensite fraction is considered with a maximum martensite fraction of 0.5
- Initially, multiple variants of the martensite phase co-existed with austenite. After a full loading cycle, since T > A_f and the martensite cannot exist at these temperatures, the wire is completely transformed into austenite. This induces a negative strain in the wire, resulting in shrinkage of its length.

Finite Element Simulations



End displacement ($A_s < T < A_f$)



End displacement ($M_s < T < A_s$)

Concluding Remarks

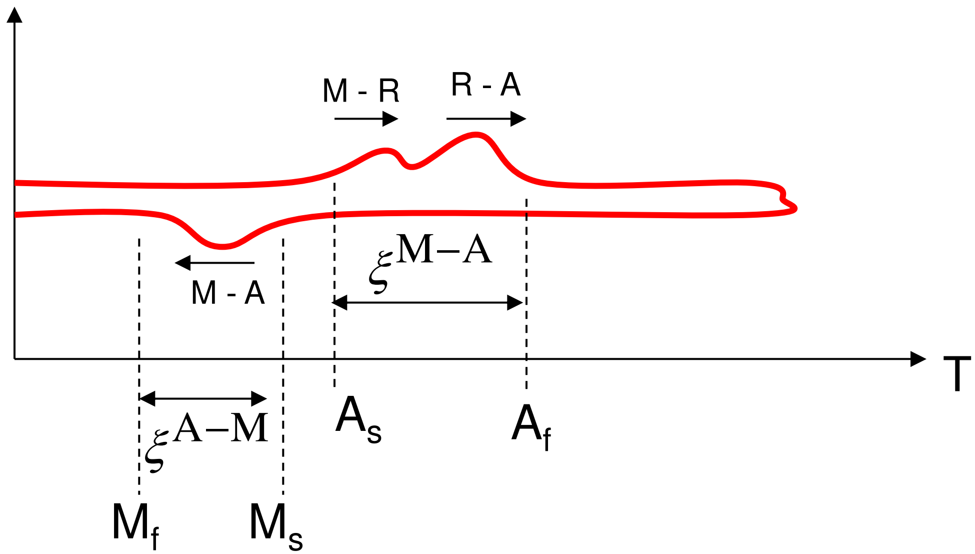
- Martensite evolution takes place due to the interplay between thermo-mechanical forces and interface driving forces. To investigate the effect of microstructural inhomogeneity on these forces and consequently on martensite evolution, scaling parameters have been introduced

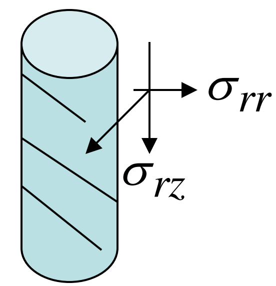
- A finite element model of SMA wire based on incremental form of the one-dimensional constitutive equation in incorporated using an updated lagrangian formulation to solve the boundary value problem.

- Inclusion of rate dependent evolution aspects and the microstructural inhomogeneity play a crucial roles in control response. SMA wire can be effective to accomplish control of buckling type devices provided the inhomoheneity effects are stabilized. The developed procedure can be employed to arrive at such stabilization process.

Acknowledgements

- General Motors
  (Enhanced SMA Material Model for Actuators)





$$\sigma_{zz} = \sigma_{zz}(z, r)$$

- Aeronautical Development Agency (ADA), Bangalore, India
  (SMA based morphing aircraft program)