Experimental Studies Toward the Development of a Structural Health Monitoring System Using Damage Force Indicator

A.K. Rao[a], D.P.K Muthukumaraswamy[a], D. Roy Mahapatra[b]
S. Gopalakrishnan[b] and M.S. Bhat[a]

[a]Dynamics and Control Laboratory
[b]ARDB Center for Composite Structures
Department of Aerospace Engineering
Indian Institute of Science

Overview

- Wave propagation, principles and techniques, their applications in structural health monitoring

- Computational paradigm: Spectra vs. Boundary Value Problem (BVP)
  How small is the wavelength or how large is the structure?

- A Damage Force Indicator (DFI) approach:

  Spectral properties + BVP + Sensor Network

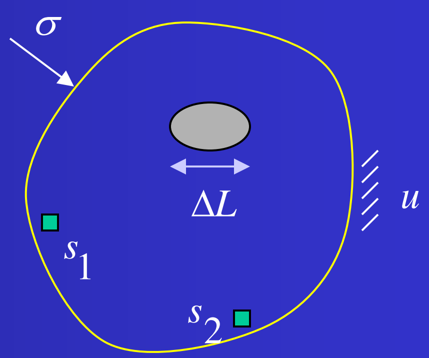
  The issue is now how large is the sensor network: A measurement paradigm

- Experimental demonstration

Wave propagation, principles and techniques, their applications in structural health monitoring

Wave problem $f(\alpha\partial_t, \beta\partial_{\mathbf{x}}, \mathbf{u}) = \mathbf{0}$

$$u = ae^{-i(\mathbf{k}.\mathbf{x}-\omega t)} \qquad \forall k \in g(\omega, \alpha, \beta)$$



(a) Interior-Exterior Scattering

$$\Delta L << \mathrm{Min}(\lambda_g)$$

$$\lambda_g = \frac{c_g}{\nu}, \qquad c_g = \frac{d\omega}{dk}, \qquad \nu = \frac{\omega}{2\pi}$$

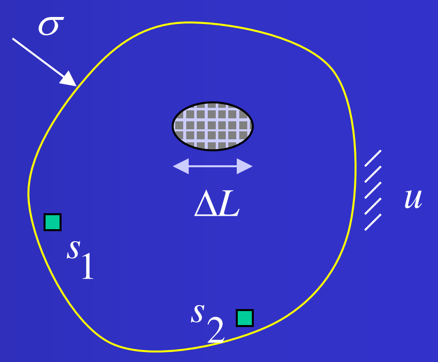
(b) Vibration

$$\Delta L \le \frac{1}{2}\mathrm{Min}(\lambda_g)$$

Boundary Value Problem (BVP)

$$\delta\Pi = \int_\Omega \bar{\mathbf{u}}.f(\alpha\partial_t, \beta\partial_{\mathbf{x}}, \mathbf{u})\mathrm{d}\mathbf{x} + \int_\Gamma \bar{\mathbf{u}}.b(\alpha'\partial_t, \beta'\partial_{\mathbf{x}}, \mathbf{u})\mathrm{d}s = 0 \qquad \mapsto \qquad a:\{u = ae^{-i(\mathbf{k}.\mathbf{x}-\omega t)}\}$$

Computational paradigm: Spectra vs. Boundary Value Problem
How small is the wavelength or how large is the structure?



Spectral expansion

$$u(x,t) = ae^{-i(k.x-\omega t)} \equiv a\left[1 - i(kx-\omega\tau) + \frac{1}{2}i^2(kx-\omega\tau)^2 - \ldots\right]$$

h-p interpolation in BVP

$$u_m(x,t) = \sum_{j=0}^{p} a'_{mj}(t)x^p \qquad x \in \Omega_m \qquad m >> p$$

where let *m* be the number of discretization in *x* with uniform element size *h*

$$O(x) = O(mh) = O(m\Delta L)$$

Let μ be the error 1 norm in computed u by solving BVP using h-p FE. The system size is given by

$$M = A_{sf}m^3$$

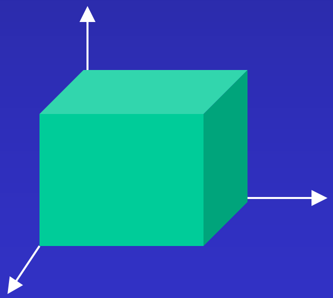
Computational paradigm: Spectra vs. Boundary Value Problem (BVP)

How small is the wavelength or how large is the structure?

Equivalently, the truncation in the spectral expansion which corresponds to the discretization in h-p gives

$$\omega \leq \frac{[\mu(n+1)!]^{1/(n+1)}}{\left(\frac{m\Delta L}{c} - \tau\right)}$$

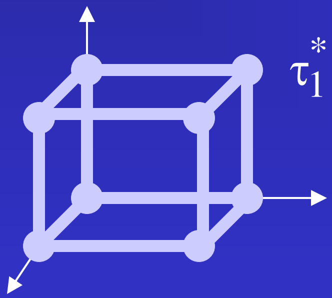
Solving wave propagation in 3D large scale structures using spectral FE



$$\rho\ddot{\mathbf{u}} - \nabla\boldsymbol{\sigma} = \mathbf{0}$$

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 c^2$$

$$\tau_1 = \frac{1}{\omega}[l_x k_x + l_y k_y + l_z k_z]$$



$$\tau_1^* = [l_x \frac{d\bar{k}_x}{d\omega} + l_y \frac{d\bar{k}_y}{d\omega} + l_z \frac{d\bar{k}_z}{d\omega}]$$

$$\delta\prod + \delta\frac{1}{2}\sum \alpha(\tau_1 - \tau_1^*)^2 \dot{\mathbf{u}}^2 = 0$$

Spectral Finite Element Method

Transformation of **u** and **f** from time domain to frequency domain:

$$\mathbf{u}(x,t)=\sum_{n=1}^{N}\hat{\mathbf{u}}(x,\omega_n)e^{i\omega_n t}=\sum_{n=1}^{N}\left(\sum_{j}\tilde{\mathbf{u}}_j e^{-ik_j x}\right)e^{i\omega_n t}, \qquad \mathbf{f}(x,t)=\sum_{n=1}^{N}\hat{\mathbf{f}}(x,\omega_n)e^{i\omega_n t}$$

Characteristic system: $\mathbf{F}(k_j,\omega_n)\tilde{\mathbf{u}}_j=\mathbf{0}$ (m × m)
m = no. of wave equations

Roots k_j , j=1,…,J : $\mathrm{Det}\mathbf{F}(k_j,\omega_n)=0$

Generic field vector :

$$\hat{\mathbf{u}}(x,\omega_n)_{(m\times 1)}=\mathbf{R}_{(m\times J)}\mathbf{\Lambda_0}(k_j,x)_{(J\times J)}\tilde{\mathbf{u}}_{(J\times 1)}$$
$$=\mathbf{T_1}(x,\omega_n)\tilde{\mathbf{u}}$$

Nodal vector of field variables :

$$\hat{\mathbf{u}}^e=\begin{bmatrix}\mathbf{T_1}(0,\omega_n)\\ \mathbf{T_1}(L,\omega_n)\end{bmatrix}\tilde{\mathbf{u}}=\mathbf{T_2}\tilde{\mathbf{u}}$$

Spectral Finite Element Method

Field interpolation : $$\hat{\mathbf{u}}(x,\omega_n) = \mathbf{T_1}(x,\omega_n)\mathbf{T_2}^{-1}\hat{\mathbf{u}}^e = \aleph(x,\omega_n)^e\,\hat{\mathbf{u}}^e$$

Force field: $$\hat{\mathbf{f}}(x,\omega_n) = \mathbf{Q_0}\hat{\mathbf{u}}(x,\omega_n) + \mathbf{Q_1}\frac{\partial}{\partial x}\hat{\mathbf{u}}(x,\omega_n) + \cdots$$

Nodal force vector:

$$\hat{\mathbf{f}}^e = \begin{bmatrix} -\left(\mathbf{Q_0 R \Lambda_0} + \mathbf{Q_1 R \Lambda_1}\right)_{x=0} \\ \left(\mathbf{Q_0 R \Lambda_0} + \mathbf{Q_1 R \Lambda_1}\right)_{x=L} \end{bmatrix} \mathbf{T_2}^{-1}\hat{\mathbf{u}}^e = \hat{\mathbf{K}}^e\hat{\mathbf{u}}^e$$

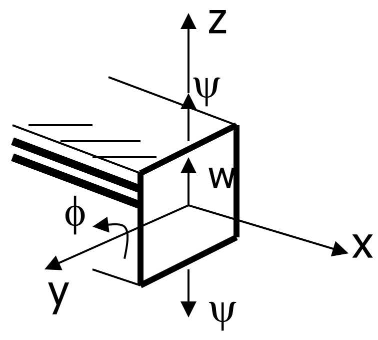
The assembled finite element system is solved at each ω_n , n=1,…,N, where N is the Nyquist frequency. Time domain fields (u,f,ε,σ) are computed by inverse FFT of the corresponding spectral amplitudes.

Axial-Flexural-Shear Coupled Waves in Composite

Kinematics:

$$u(x,y,z,t)=u^o(x,t)-z\phi(x,t)$$

$$w(x,y,z,t)=w(x,t)+z\psi(x,t)$$



Constitutive model:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{zz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{13} & 0\\ \bar{Q}_{13} & \bar{Q}_{33} & 0\\ 0 & 0 & \bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{zz}\\ \gamma_{xz}\end{Bmatrix}$$

Wave equations:

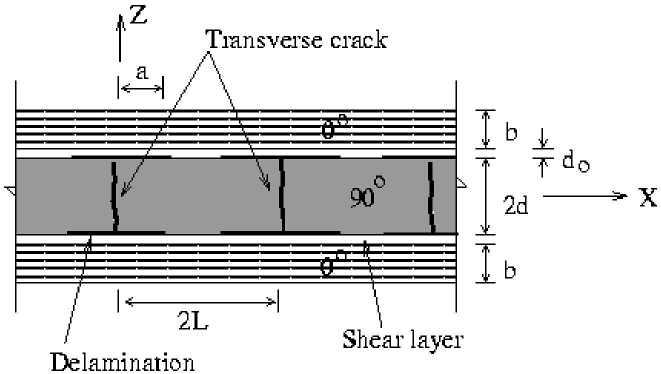
$$\delta u^o:\quad I_0\ddot{u}^o-\underline{I_1\ddot{\phi}}-A_{11}u^o_{,xx}+\underline{B_{11}\phi_{,xx}}-A_{13}\psi_{,x}=0$$

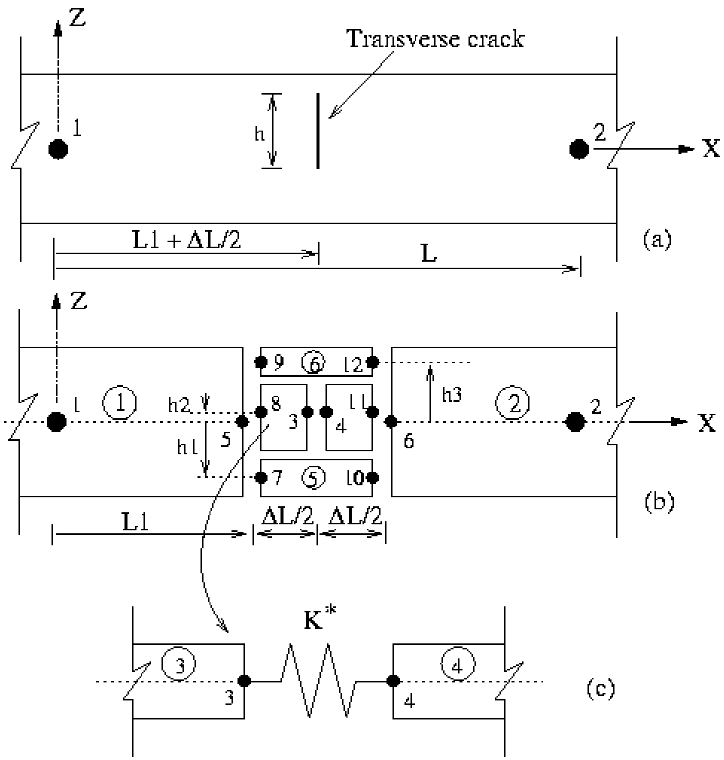
$$\delta\psi:\quad I_2\ddot{\psi}+\underline{I_1\ddot{w}}+A_{13}u^o_{,x}\underline{-B_{13}\phi_{,x}}+A_{33}\psi\underline{-B_{55}(w_{,xx}-\phi_{,xx})}-D_{55}\psi_{,xx}=0$$

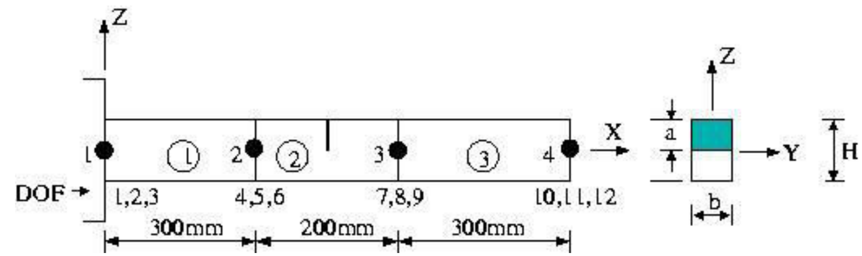
$$\delta w:\quad I_0\ddot{w}+\underline{I_1\ddot{\psi}}-A_{55}(w_{,xx}-\phi_{,x})\underline{-B_{55}\psi_{,xx}}=0$$

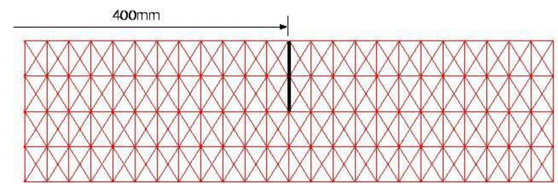
$$\delta\phi:\quad I_2\ddot{\phi}\underline{-I_1\ddot{u}^o}-A_{55}(w_{,x}-\phi)\underline{-B_{55}\psi_{,x}+B_{11}u^o_{,xx}}-D_{11}\phi_{,xx}\underline{+B_{13}\psi_{,x}}=0$$

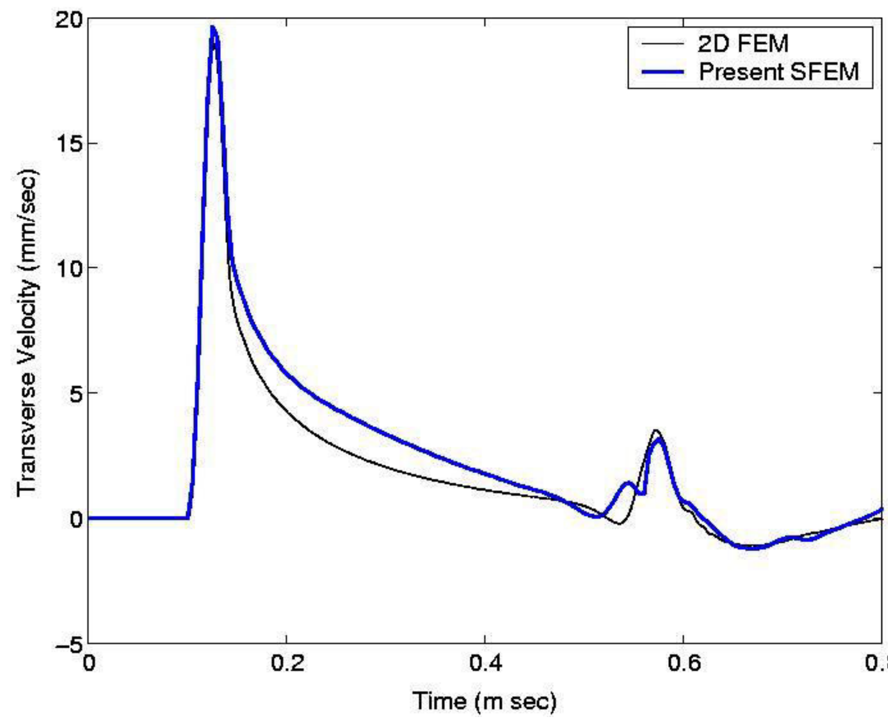
Surface-breaking cracks



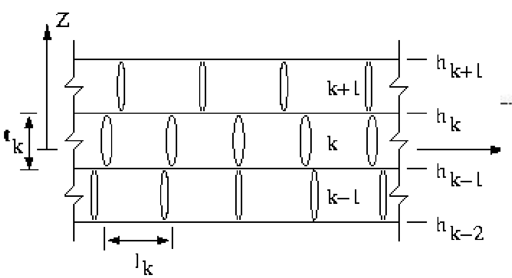


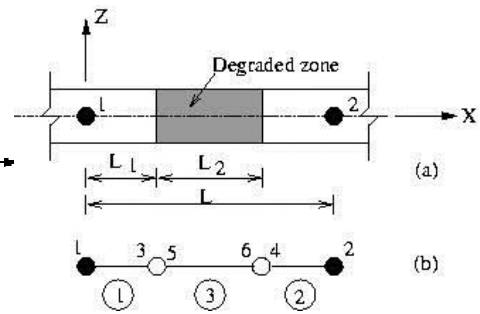






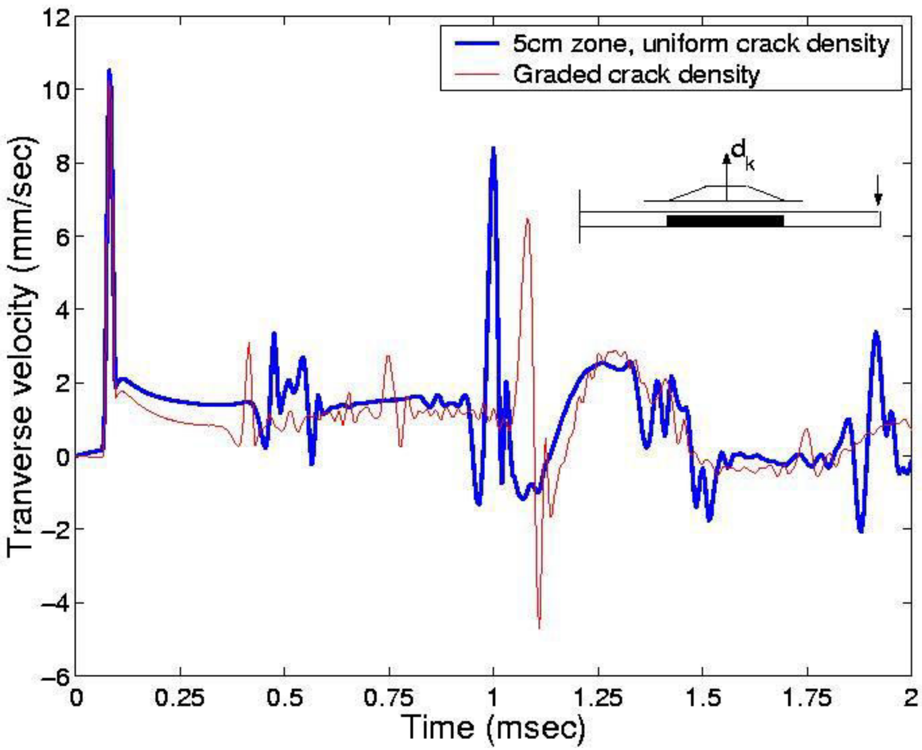
Matrix cracks

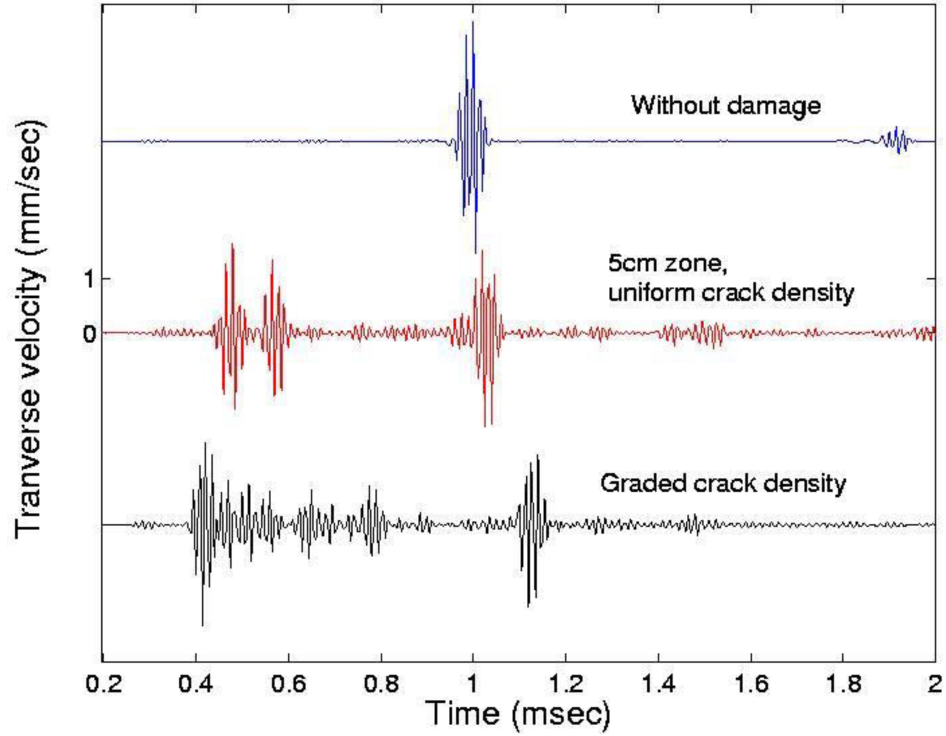




$$W_{(c)} = \frac{1}{2}\left(\boldsymbol{\varepsilon}^T \mathbf{C}\boldsymbol{\varepsilon} - \Delta W\right) + \sum_{k=1}^{M} f_k\left(\boldsymbol{\sigma}_{k(R)}\right)$$

$$\begin{Bmatrix} N_x \\ M_x \\ V_x \end{Bmatrix} = \begin{bmatrix} \alpha_{11}C_{EE} & \beta_{11}C_{EB} & 0 \\ \beta_{11}C_{EB} & \lambda_{11}C_{BB} & 0 \\ 0 & 0 & \alpha_{55}C_{SS} \end{bmatrix} \begin{Bmatrix} u^o,_x \\ \phi,_x \\ w,_x + \phi \end{Bmatrix}$$





Damage Force Indicator (DFI) approach:

Spectral properties + BVP + Sensor Network

Spectral Finite Element

Reduced-order model

DFI and Real-time Estimates

Baseline structure

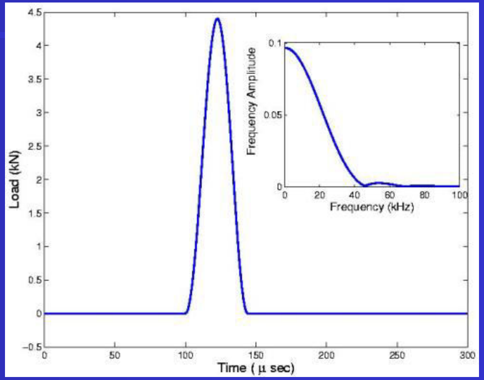
$$
\begin{bmatrix}
\overline{\mathbf{K}}_{11} & \overline{\mathbf{K}}_{1p} & \mathbf{0} & \mathbf{0} \\
\overline{\mathbf{K}}_{p1} & \overline{\mathbf{K}}_{pp} & \overline{\mathbf{K}}_{pq} & \mathbf{0} \\
\mathbf{0} & \overline{\mathbf{K}}_{qp} & \overline{\mathbf{K}}_{qq} & \overline{\mathbf{K}}_{q2} \\
\mathbf{0} & \mathbf{0} & \overline{\mathbf{K}}_{2q} & \overline{\mathbf{K}}_{22}
\end{bmatrix}_{(m'\times n')}
\begin{Bmatrix} \hat{\mathbf{u}}_1 \\ \hat{\mathbf{u}}_p \\ \hat{\mathbf{u}}_q \\ \hat{\mathbf{u}}_2 \end{Bmatrix}
=
\begin{Bmatrix} \hat{\mathbf{f}}_1 \\ \mathbf{0} \\ \mathbf{0} \\ \hat{\mathbf{f}}_2 \end{Bmatrix}
$$

Structure with damage

$$\Delta\hat{\mathbf{f}}(\omega_n) = \hat{\mathbf{K}}_h(\omega_n)\hat{\mathbf{u}}_d(\omega_n) - \boxed{\hat{\mathbf{f}}_d(\omega_n)} \qquad \forall n \in [n_1, n_2]$$

$$\hat{\mathbf{u}}_d = \hat{\mathbf{u}}_h + \boxed{\Delta\hat{\mathbf{u}}} \qquad\qquad \hat{\mathbf{R}}(\omega_n) = \Delta\hat{\mathbf{f}}\Delta\hat{\mathbf{f}}^*$$

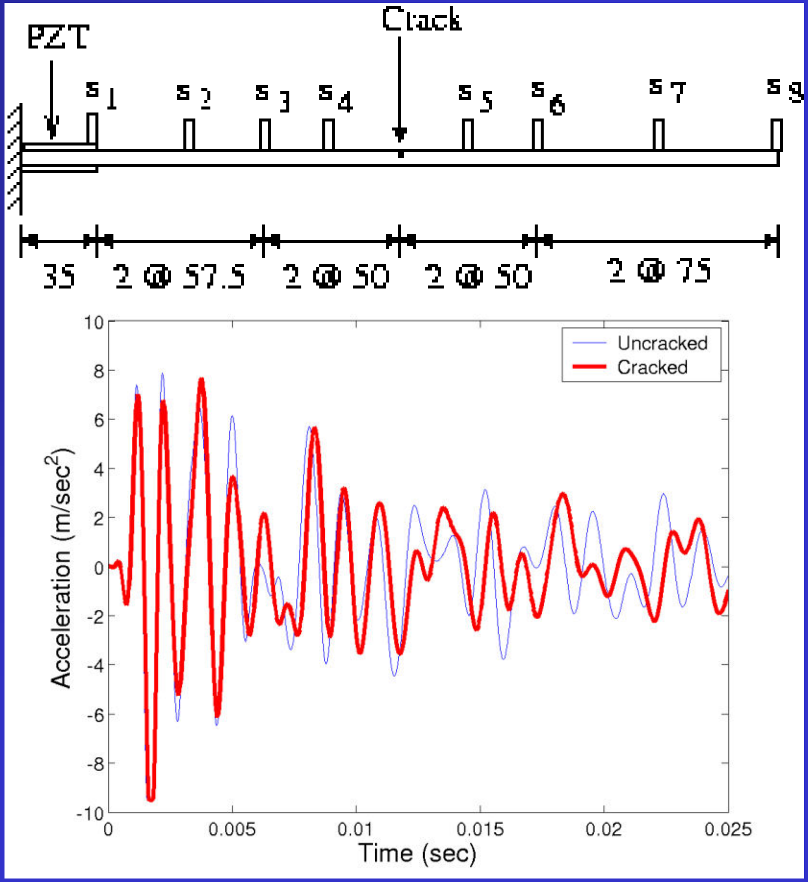
Damage Force Indicator (DFI) approach:

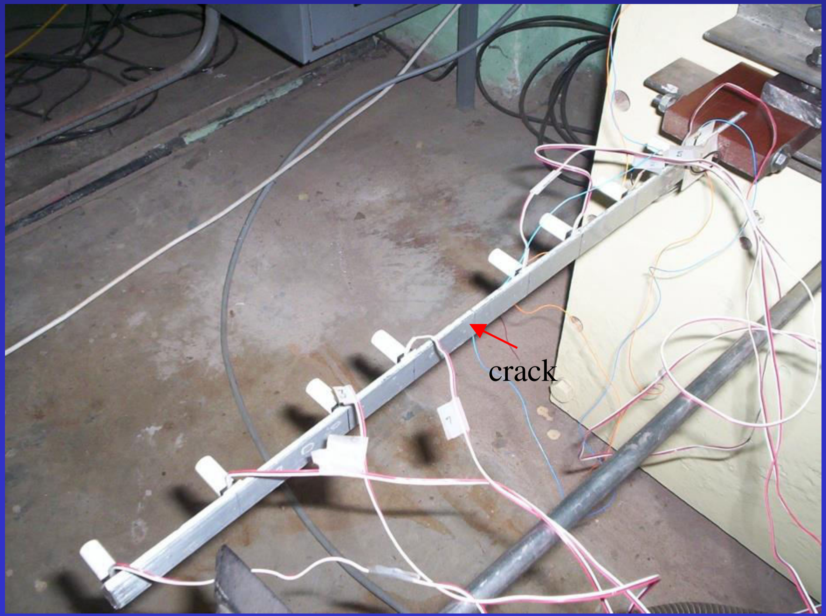


$$\hat{\mathbf{R}}(\omega_n) = \Delta\hat{\mathbf{f}}\Delta\hat{\mathbf{f}}^*$$

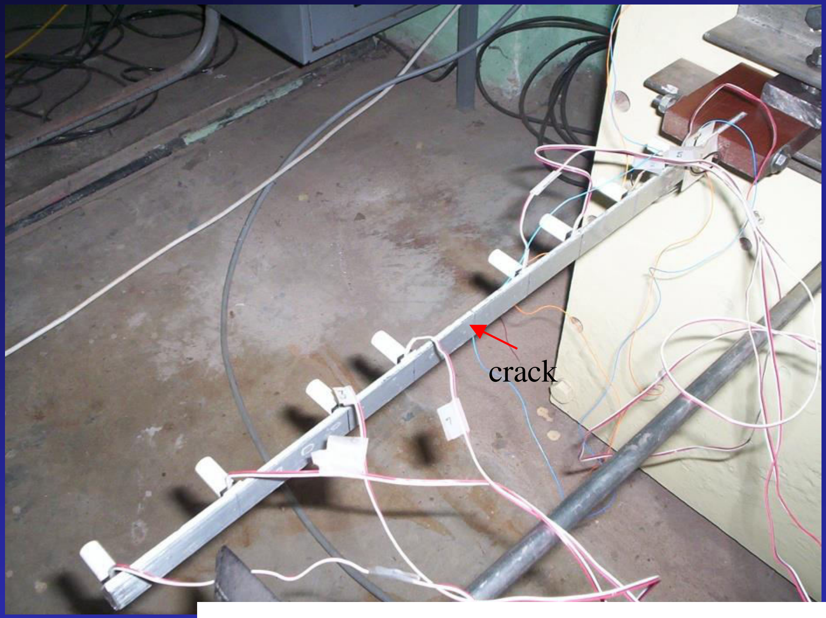
$$d_i = \sum_n \left|\hat{R}_{ii}\right|, \qquad i \in [1, m], \qquad n \in [n_1, n_2]$$

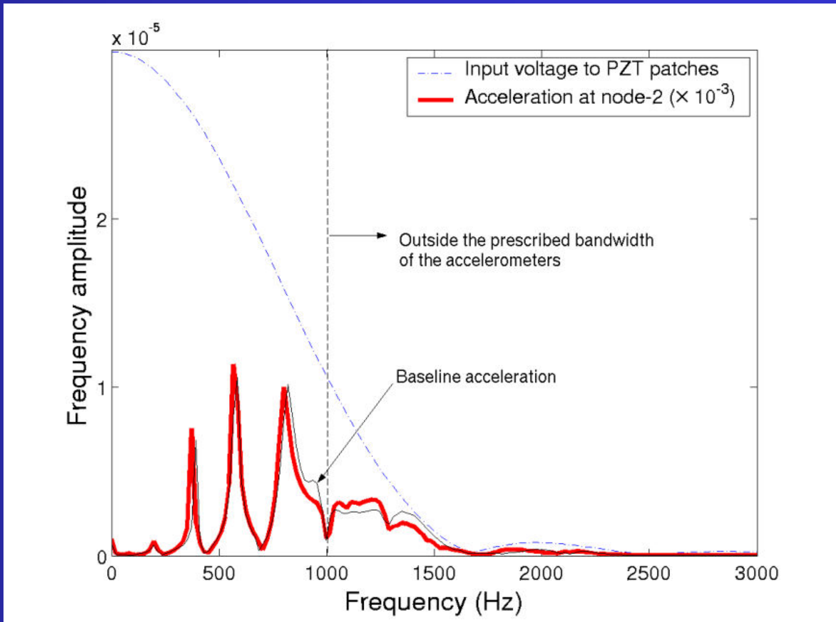
DFI $$\bar{d}_i = \frac{d_i}{d_m} - 1$$

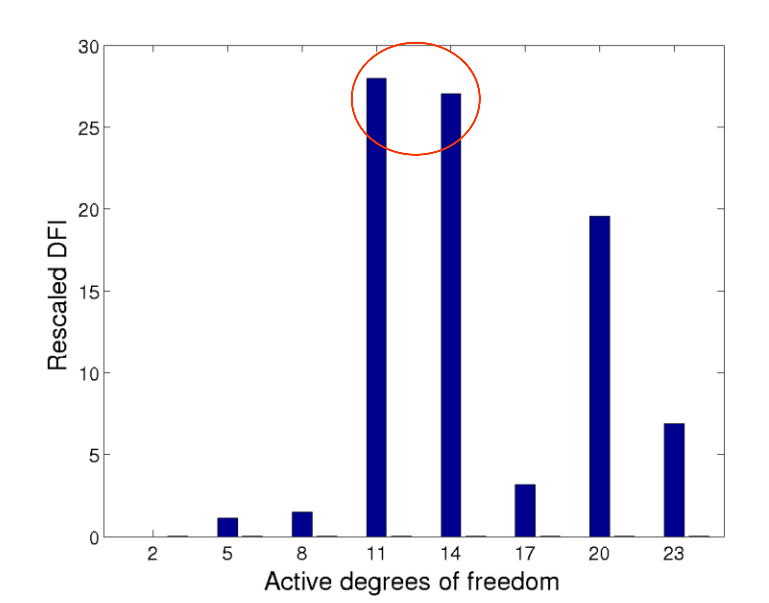


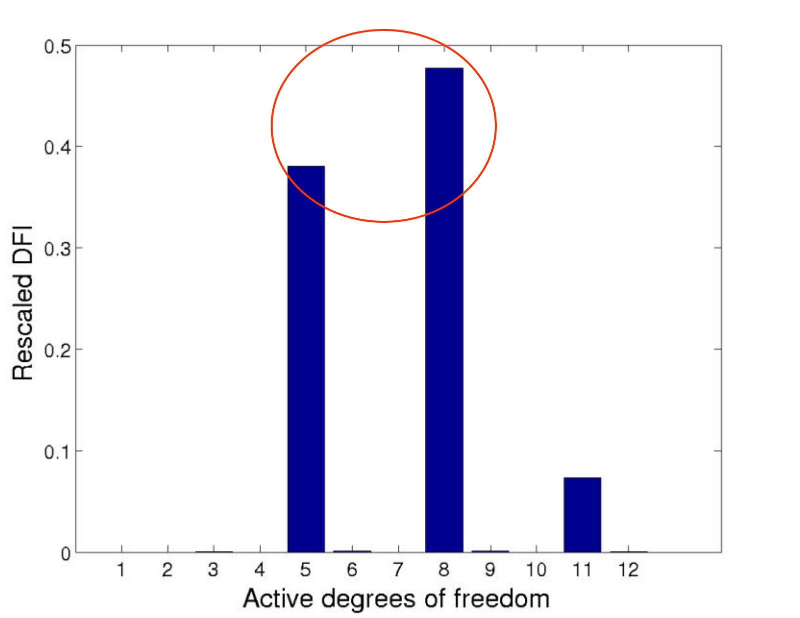


Experimental demonstration

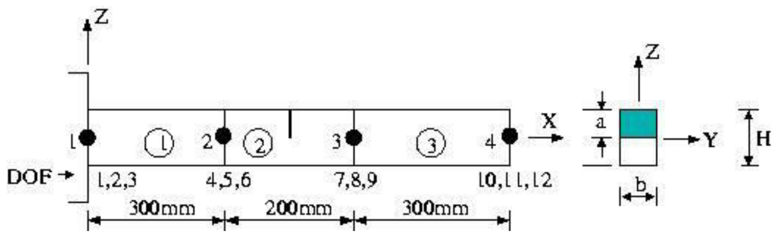




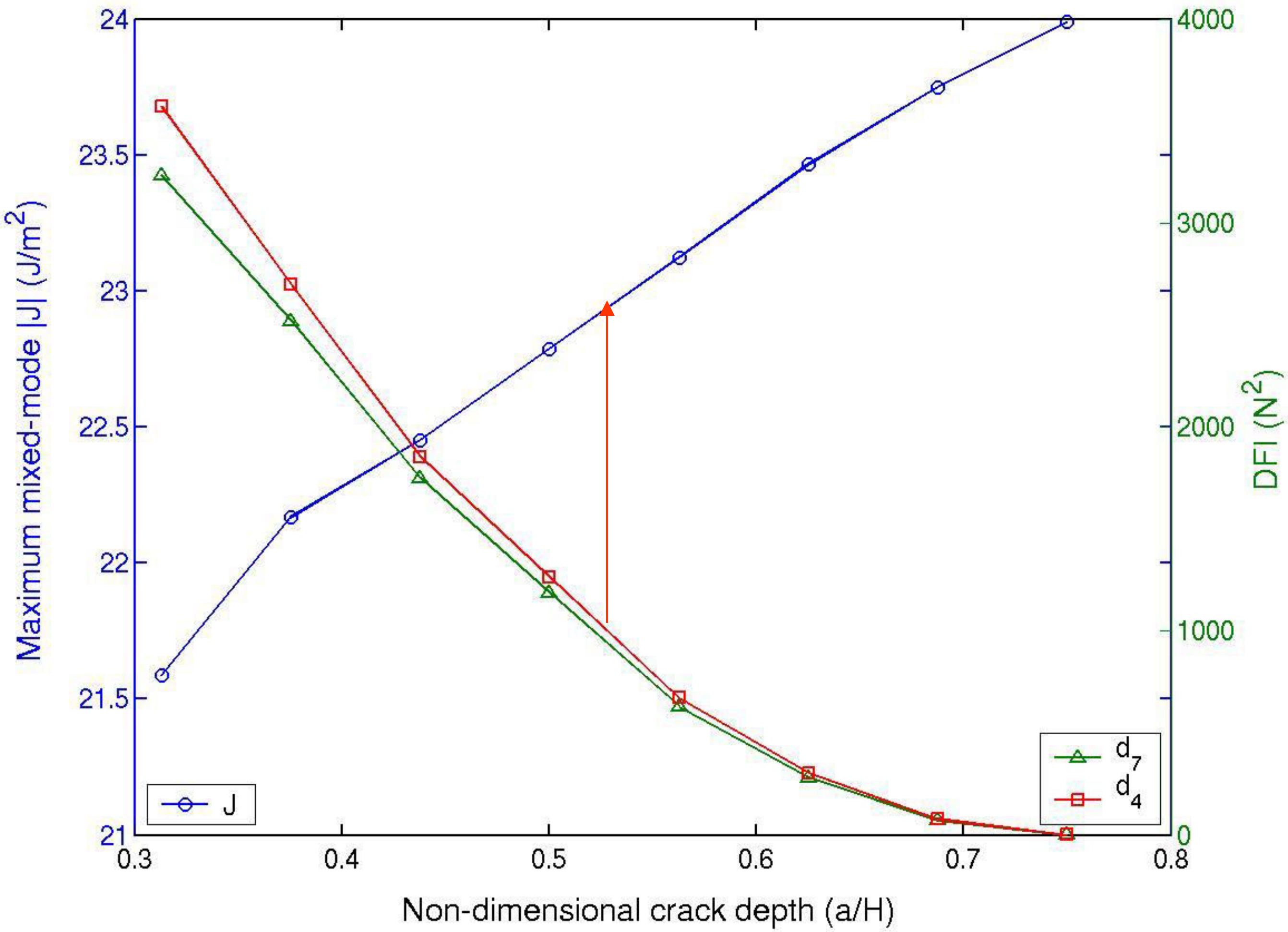




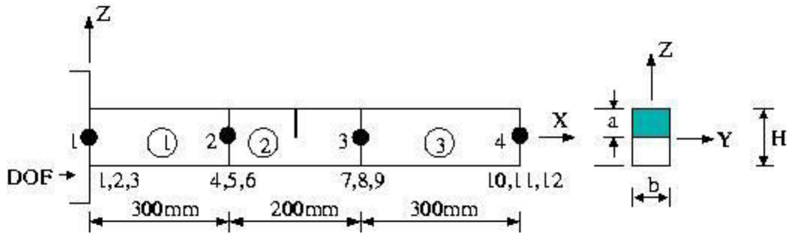
Quantitative NDE



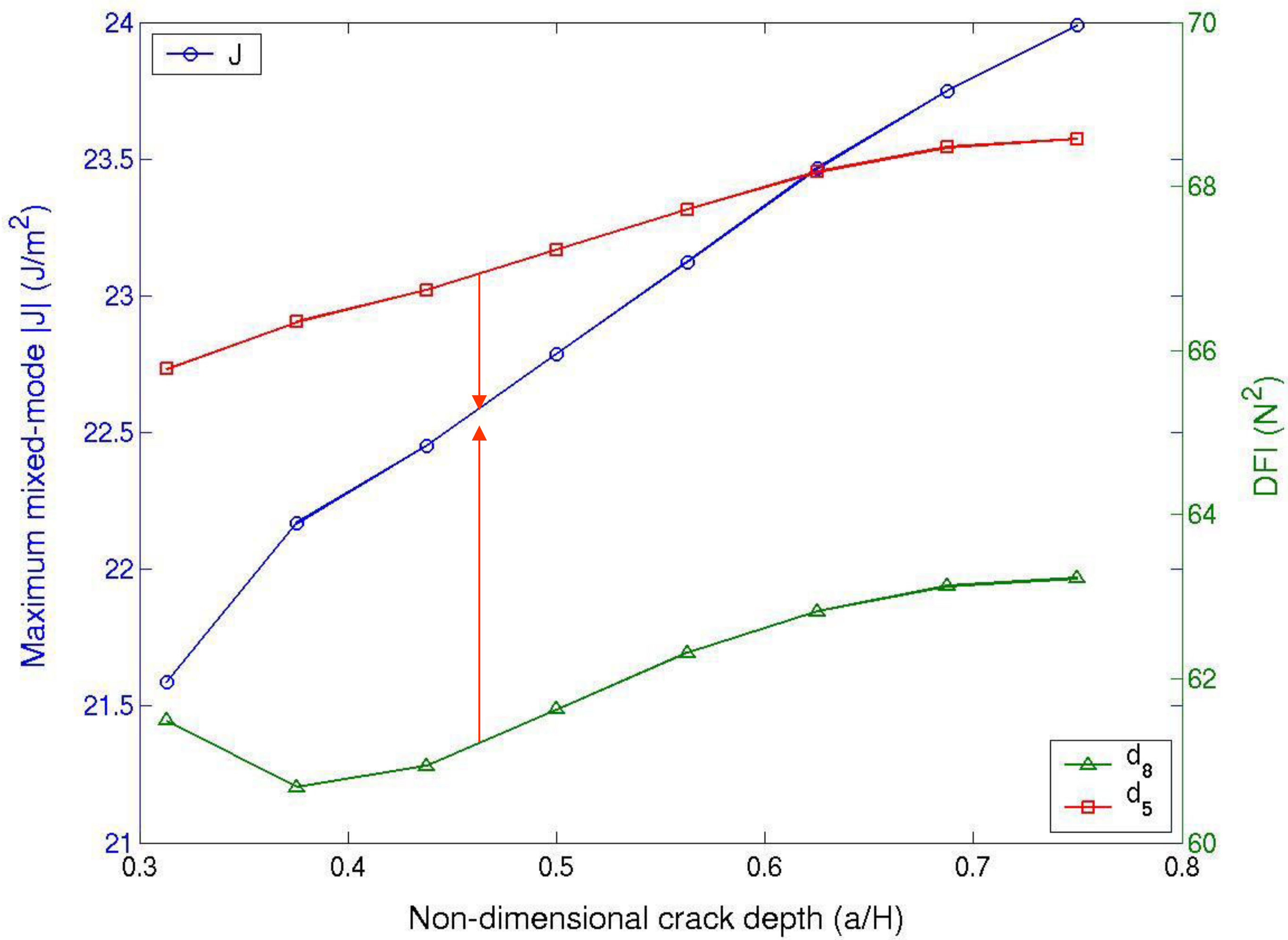
SERR-DFI curve



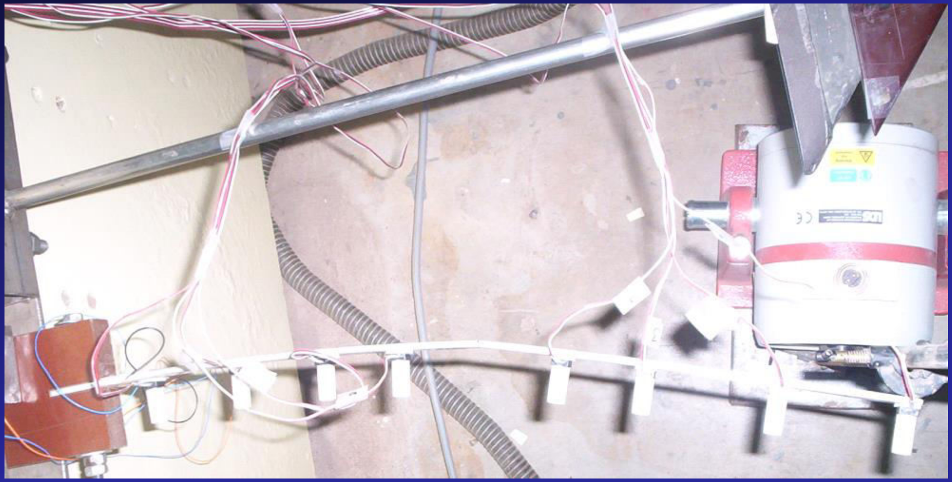
Quantitative NDE

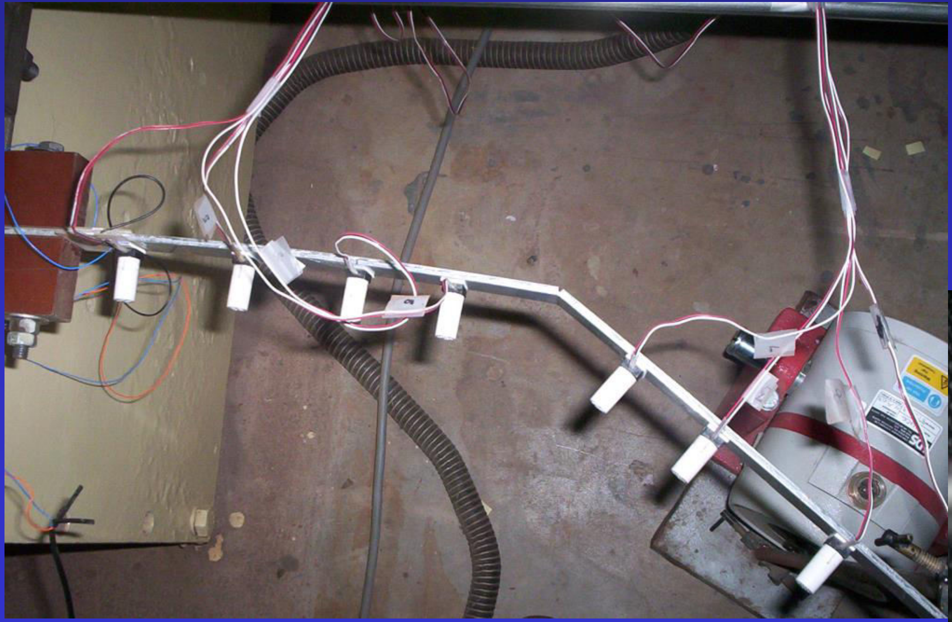


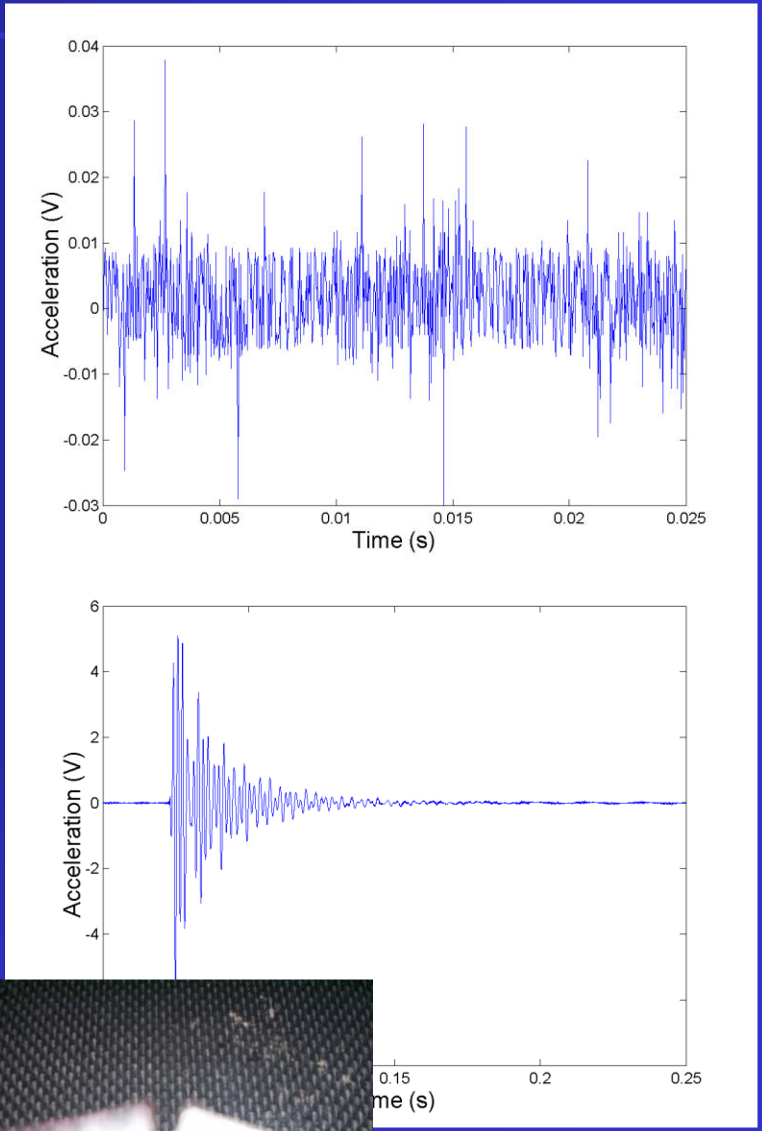
SERR-DFI curve



Progressive failure monitoring







Conclusions

- A Damage Force Indicator (DFI) approach is developed by which the location and severity of damage can be monitored without actually knowing the detail onset of the damage or modeling it.
- The method uses the dynamic stiffness matrix of the healthy global or substructure as model input. This matrix must adequately include the wave propagation characteristics over the frequency band of measurement.
- The model can accommodate significant amount of uncertainty in the boundary conditions as well as in measurement
- The approach does not require measurement of the source disturbances which act as the diagnostic signals.
- Experimental demonstration shows that real-time monitoring of progressive failure is possible.
- A real system development would require a fault tolerant network of MEMS sensors and actuators