Design of Cellular Composite for Vibration and Noise Control: Effects of Inclusions and Filling Patterns on Acoustic Bandgap

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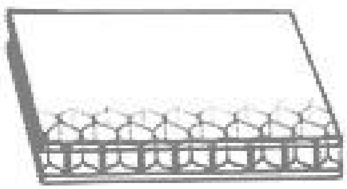
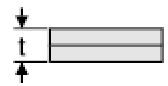
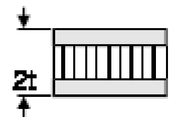
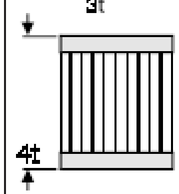
Introduction

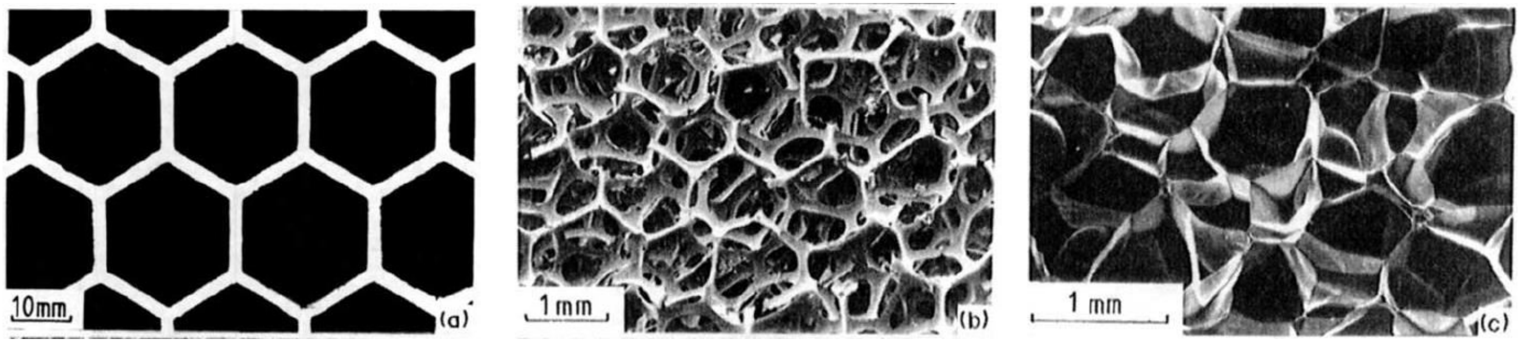


- What is a Cellular solid?
  - It is one made up of an interconnected network of solid struts or plates
- What are applications of cellular solids?
  - Noise control
  - Thermal Insulation
  - Packaging
  - Structural, impact resistant
  - Light weight and other applications

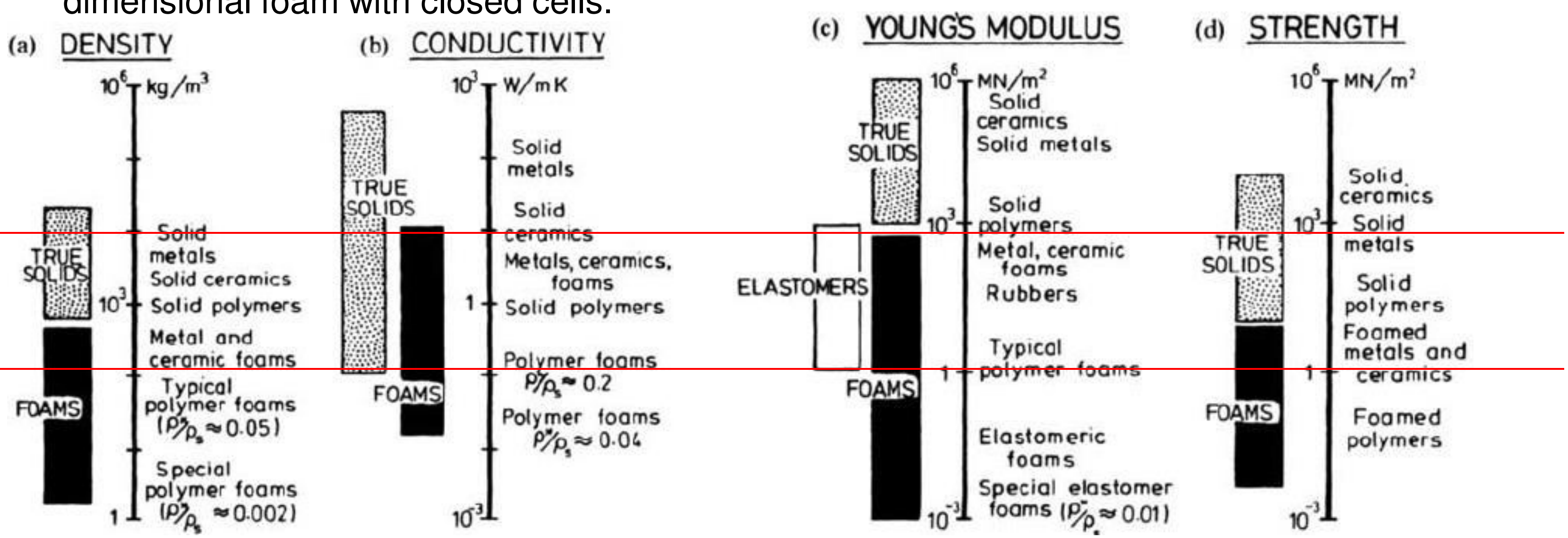
	Solid Material	Core Thickness t	Core Thickness 3t
			
Stiffness	1.0	7.0	37.0
Flexural Strength	1.0	3.5	9.2
Weight	1.0	1.03	1.06

- What is a sandwich panel?
- What are the advantages of honeycomb sandwich panels?
  - Honeycomb sandwich panels offer outstanding stiffness and strength with low weight
  - The cellular structures can be patterned in various forms to achieve very good noise screening properties

Typical structures of cellular solids



(a) A two dimensional honeycomb, (b) A three dimensional foam with open cells, (c) A three dimensional foam with closed cells.



(a) Density (b) Thermal conductivity (c) Young's modulus (c) Compressive strength

- What are the applications of sandwich panel?
  - Aircraft flooring
  - Aircraft interiors
  - Tooling industry
  - Ship Interiors
  - Train interiors
  - Construction Industry

- What are problems and issues in designing a sandwich panel?
  - The core material of the sandwich panel is a cellular material, and the material properties of core material are distributed non-uniformly in space. Because of the non-uniformity, modeling of material properties of sandwich panels is a challenging task.

  - Because the core material is attached with two thin stiff face sheets on top and bottom, there will be entrapped air columns within the cell cavities. Therefore there is a chance of local resonance in the cell walls. To consider the effect of this entrapped air on vibration and acoustic behavior of sandwich panels is quite challenging.

Objectives

- Developing a modeling technique for cellular composite materials.
- Analysis of wave dispersion in a cellular composite laminate with spatially modulated microstructure to determine the behavior of wave propagation through the structure, and for analyzing stop band and pass band patterns.
- In the wave dispersion analysis to get better understanding, we need to have effective properties of cellular material, therefore another objective is static homogenization of honeycomb panel.
- To determine the dynamic response of the sandwich panel under an external load considering effect of cell wall pressure.

Background Literature

Basic principle of disturbance suppression by Cellular composite materials :

- Cellular composites have microstructures, which deform very differently than the elastic continua.
- Due to the geometric features at multiple scales, certain resonance characteristics (often due to the highly flexible cell walls /ligaments) are observed in the microstructure.
- One approach in modeling is that of an effective homogenized medium through which elastic wave propagates. The effective properties are assumed to be varying as a function of the spatio-temporal parameters. This is known as dynamic material. (Suzanne L. Weekes, "Dispersion effects in dynamic laminates", *Physica B* 338(2003), 64-66.)

Background Literature

- Analysis of wave propagation by assuming dynamic stiffness modulation has been reported by Sorokin and Grishina. (Konstantin, A. Lurie, 1997 “Effective properties of smart elastic laminatess and the screening phenomenon”, *International Journal of solids and structures* Vol. 34, No.13, 1633-1643).

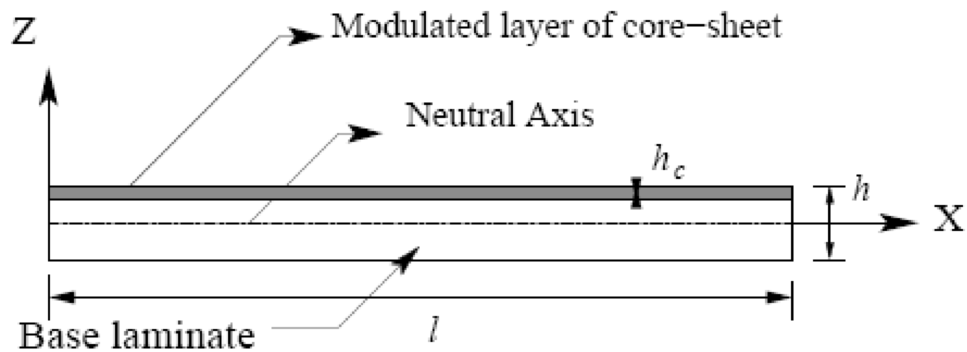
- Locally resonant band-gap phenomenon (as an extension to Helmholtz resonator cavities) have been analyzed by Wang and Ivansson. (Wang.G, Wen.X, Wen.J and Liu.Y, “Quasi-one-dimensional periodic structure with locally resonant band gap”, *Journal of Applied Mechanics* 73, 176 (2006)).

- Damping of thin-walled honeycomb structures using energy absorbing foam was studied by Woody. (Shane C. Woody, Stuart T. Smith 2006, “Damping of a thin-walled honeycomb structure using energy absorbing foam”, *Journal of sound and vibration* 291, 491-502).

Wave dispersion analysis in parametrically modulated laminate beam

- The cellular composite beam is modeled as a beam with layer-wise parametrically modulated material properties.

  Cellular laminate configuration:



- In view of the various possible modes of deformations of the individual cell walls, a higher-order beam theory is considered.

Kinematics

- Displacement field:

$$u(x,z,t) = u^0(x,t) + z\phi(x,t) + c_0 z^3\left(\phi(x,t) + \frac{\partial w(x,t)}{\partial x}\right) \;,\; w(x,z,t) = w^0(x,t).$$

- For traction free boundary condition on the surface

$$c_0 = -4/3h^2.$$

- Strain field:

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u^0}{\partial x} + z\frac{\partial \phi}{\partial x} + c_0 z^3\left(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w^0}{\partial x^2}\right) \;, \quad \epsilon_{yy} = \epsilon_{zz} = 0 \;,$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi + 3c_0 z^2\left(\phi + \frac{\partial w^0}{\partial x}\right) + \frac{\partial w^0}{\partial x} \;, \quad \gamma_{xy} = \gamma_{yz} = 0 \;.$$

Constitutive modeling using parametric representation

- The properties of cellular solids such as density and stiffness are distributed non-uniformly in space. Therefore they have to be modeled with some functions, which can describe this kind of pattern.
- Constitutive model used for base laminate and cellular layer:

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & 0 \\ 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} , \quad \begin{Bmatrix} \sigma_{xx_p} \\ \tau_{xz_p} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11_p} & 0 \\ 0 & \bar{Q}_{55_p} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx_p} \\ \gamma_{xz_p} \end{Bmatrix}$$

- Cellular layer properties:

$$\bar{Q}_{11_p}(x,t) = \bar{Q}_{11}\left[1 + \alpha_1 \sin\left(\frac{2\pi x}{\lambda_m} + \theta\right)\right],$$

$$\bar{Q}_{55_p}(x,t) = \bar{Q}_{55}\left[1 + \alpha_2 \sin\left(\frac{2\pi x}{\lambda_m} + \theta\right)\right],$$

$$\rho_p(x,t) = \rho\left[1 + \alpha_3 \sin\left(\frac{2\pi x}{\lambda_m} + \theta\right)\right].$$

- Governing equations of motion in the cellular laminate are obtained by using Hamilton's principle.
- Hamilton's principle:

$$\delta \int_{t_1}^{t_2} (\Gamma - U)\, dt = 0$$

- Kinetic energy (Γ) and Strain energy (U) :

$$\Gamma = \int_0^l \int_A \left(\frac{1}{2}\rho \dot{u}^2 + \frac{1}{2}\rho \dot{w_0}^2 \right) dA dx \ ,$$

$$U = \int_0^l \int_A \left(\frac{1}{2}\sigma_{xx}\epsilon_{xx} + \frac{1}{2}\tau_{xz}\gamma_{xz} \right) dA dx$$

Wave equations:

- In plane wave equation:

$$\delta u^0 : -I_0\left[1+f(\lambda_m,\theta)\right]\ddot{u}^0-(I_1+c_0I_3)\left[1+f(\lambda_m,\theta)\right]\ddot{\phi}-$$

$$c_0I_3\left[1+f(\lambda_m,\theta)\right]\frac{\partial\ddot{w}}{\partial x}+A_{11}f'(\lambda_m,\theta)\frac{\partial u^0}{\partial x}+(B_{11}+c_0F_{11})f'(\lambda_m,\theta)\frac{\partial\phi}{\partial x}+$$

$$c_0F_{11}f'(\lambda_m,\theta)\frac{\partial^2 w}{\partial x^2}+A_{11}\left[1+f(\lambda_m,\theta)\right]\frac{\partial^2 u^0}{\partial x^2}+(B_{11}+c_0F_{11})$$

$$\left[1+f(\lambda_m,\theta)\right]\frac{\partial^2\phi}{\partial x^2}+c_0F_{11}\left[1+f(\lambda_m,\theta)\right]\frac{\partial^3 w^0}{\partial x^3}=0.$$

- Used Integral quantities:

$$I_i=\int\rho z^i dz\ , i=1,2,...6$$

$$A_{ii}=\int\bar{Q}_{ii}dz\ ,\quad B_{ii}=\int\bar{Q}_{ii}z\,dz\ ,\quad D_{ii}=\int\bar{Q}_{ii}z^2dz\ ,\quad F_{ii}=\int\bar{Q}_{ii}z^3dz$$

$$f(\lambda_m,\theta)=\alpha_j\sin\left(\frac{2\pi x}{\lambda_m}+\theta\right)\ ,\quad f'=\frac{\partial f}{\partial x}\ ,\quad f''=\frac{\partial^2 f}{\partial x^2}$$

$$H_{ii}=\int\bar{Q}_{ii}z^4dz\ ,\quad L_{ii}=\int\bar{Q}_{ii}z^6dz\ ,$$

- Flexural wave equation:

$$
\begin{aligned}
\delta w^0 :\quad & c_0 I_3 f'(\lambda_m,\theta)\ddot{u}^0 + \left(c_0 I_4 + c_0^2 I_6\right) f'(\lambda_m,\theta)\ddot{\phi} + c_0^2 I_6 f'(\lambda_m,\theta)\frac{\partial \ddot{w}^0}{\partial x} \\
& + c_0 I_3[1 + f(\lambda_m,\theta)]\frac{\partial \ddot{u}^0}{\partial x} + (c_0 I_4 + c_0^2 I_6)[1 + f(\lambda_m,\theta)]\frac{\partial \ddot{\phi}}{\partial x} + \\
& c_0^2 I_6\left[1 + f(\lambda_m,\theta)\right]\frac{\partial^2 \ddot{w}^0}{\partial x^2} - I_0\left[1 + f(\lambda_m,\theta)\right]\ddot{w}^0 \\
& + \left\{6 c_0 D_{55} f'(\lambda_m,\theta) + A_{55} f'(\lambda_m,\theta) + 9 c_0^2 H_{55} f'(\lambda_m,\theta)\right\}\phi \\
& - c_0 F_{11} f''(\lambda_m,\theta)\frac{\partial u^0}{\partial x} + \Big\{9 c_0^2 H_{55} f'(\lambda_m,\theta) + 6 c_0 D_{55} f'(\lambda_m,\theta) + A_{55} \\
& f'(\lambda_m,\theta)\Big\}\frac{\partial w^0}{\partial x} + \Big\{\left(-c_0 H_{11} - c_0^2 L_{11}\right) f''(\lambda_m,\theta) + (6 c_0 D_{55} + 9 c_0^2 H_{55} \\
& + A_{55})[1 + f(\lambda_m,\theta)]\Big\}\frac{\partial \phi}{\partial x} + \Big\{- c_0^2 L_{11} f''(\lambda_m,\theta) + (9 c_0^2 H_{55} + 6 c_0 D_{55} \\
& + A_{55})[1 + f(\lambda_m,\theta)]\Big\}\frac{\partial^2 w^0}{\partial x^2} - \Big\{2 c_0 F_{11}\frac{\partial^2 u^0}{\partial x^2} + (2 c_0 H_{11} + 2 c_0^2 L_{11})\frac{\partial^2 \phi}{\partial x^2} \\
& + 2 c_0^2 L_{11}\frac{\partial^3 w^0}{\partial x^3}\Big\} f'(\lambda_m,\theta) - c_0 F_{11}[1 + f(\lambda_m,\theta)]\frac{\partial^3 u^0}{\partial x^3} \\
& - (c_0 H_{11} + c_0^2 L_{11})[1 + f(\lambda_m,\theta)]\frac{\partial^3 \phi}{\partial x^3} - c_0^2 L_{11}\frac{\partial^4 w^0}{\partial x^4}[1 + f(\lambda_m,\theta)] .
\end{aligned}
$$

- Underlined terms are coupling terms between flexure and shear modes

- Equation of motion for cross sectional rotation (Φ):

$$\delta\phi : \quad \left[-\left(I_1 + c_0 I_3\right)\ddot{u}^0 - \left(I_2 + 2c_0 I_4 + c_0^2 I_6\right)\ddot{\phi}\right]\left[1 + f(\lambda_m, \theta)\right]$$

$$- \left(c_0 I_4 + c_0^2 I_6\right)\left[1 + f(\lambda_m, \theta)\right]\frac{\partial \ddot{w}^0}{\partial x} - \left(A_{55} + 6c_0 D_{55} + 9c_0^2 H_{55}\right)\left[1 + f(\lambda_m, \theta)\right]\phi$$

$$+ \left(B_{11} + c_0 F_{11}\right)\bar{f}'\frac{\partial u^0}{\partial x} - \left(6c_0 D_{55} + A_{55} + 9c_0^2 H_{55}\right)\left[1 + f(\lambda_m, \theta)\right]\frac{\partial w^0}{\partial x}$$

$$+ \left(D_{11} + 2c_0 H_{11} + c_0^2 L_{11}\right)\bar{f}'\frac{\partial \phi}{\partial x} + \left(B_{11} + c_0 F_{11}\right)\left[1 + f(\lambda_m, \theta)\right]\frac{\partial^2 u^0}{\partial x^2}$$

$$+ \left(c_0 H_{11} + c_0^2 L_{11}\right)\bar{f}'\frac{\partial^2 w^0}{\partial x^2} + \left(D_{11} + 2c_0 H_{11} + c_0^2 L_{11}\right)\left[1 + f(\lambda_m, \theta)\right]\frac{\partial^2 \phi}{\partial x^2}$$

$$+ \left(c_0 H_{11} + c_0^2 L_{11}\right)\left[1 + f(\lambda_m, \theta)\right]\frac{\partial^3 w^0}{\partial x^3} = 0\ .$$

- In all the wave equations, the variables f and its derivatives are functions of space variable x. These are homogenized in wavelength scale.

Homogenization in the Wave Length scale

- The function *f* and its derivatives are homogenized over half wave length (λ/2)
- $$\bar{f} = \frac{2}{\lambda}\int_0^{\lambda/2} f(\lambda_m,\theta)dx \quad = \quad \frac{2}{\lambda}\int_0^{\lambda/2} \alpha_j \sin\left(\frac{2\pi x}{\lambda_m}+\theta\right)dx$$

$$\bar{f} = \frac{\alpha_j}{\pi}\frac{\lambda_m}{\lambda}\left[-\cos\left(\frac{\pi}{\lambda_m/\lambda}+\theta\right)+\cos\theta\right]$$

Similarly, we get

$$\bar{f}' = \frac{2}{\lambda}\int_0^{\lambda/2} f'(\lambda_m,\theta)dx \quad = \quad \frac{2}{\lambda}\alpha_j\left[\sin\left(\frac{\pi\lambda}{\lambda_m}+\theta\right)-\sin\theta\right]$$

and

$$\bar{f}'' = \frac{4\pi\alpha_j}{\lambda\lambda_m}\left[-\sin\left(\frac{\pi}{\lambda_m/\lambda}+\theta\right)+\sin\theta\right]$$

Harmonic wave approximation:

$$u^0(x,t) = \sum_n \tilde{u}(\omega) e^{i(\omega t - kx)} \; , \quad w^0(x,t) = \sum_n \tilde{w}(\omega) e^{i(\omega t - kx)} \; ,$$

$$\phi = \sum_n \tilde{\phi}(\omega) e^{i(\omega t - kx)}$$

we get the following characteristic system of equations,

$$\Big[m(\omega, \alpha, \lambda_m, \theta, k(\omega)) \Big] \begin{Bmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{\phi} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

- where

$$m_{11} = I_0(1+\bar{f})\omega^2 - iA_{11}\bar{f}'k - A_{11}(1+\bar{f})k^2 \ ,$$

$$m_{12} = -ic_0I_3(1+\bar{f})\omega^2k - c_0F_{11}\bar{f}'k^2 + ic_0F_{11}(1+\bar{f})k^3 \ ,$$

$$m_{13} = (I_1 + c_0I_3)\,(1+\bar{f})\omega^2 - i\left(B_{11}\bar{f}' + c_0F_{11}\bar{f}'\right)k - \left(B_{11} + c_0F_{11}\right)(1+\bar{f})k^2$$

$$m_{21} = -c_0I_3\bar{f}'\omega^2 + i\left(c_0I_3(1+\bar{f})\omega^2 + c_0F_{11}\bar{f}''\right)k + 2c_0F_{11}\bar{f}'k^2$$

$$-ic_0F_{11}(1+\bar{f})k^3 \ ,$$

$$m_{22} = I_0(1+\bar{f})\omega^2 + i\left(c_0^2I_6\omega^2 - 9c_0^2H_{55} - 6c_0D_{55} - A_{55}\right)\bar{f}'k +$$

$$\left(c_0^2L_{11}\bar{f}'' - \left(c_0^2I_6\omega^2 + 9c_0^2H_{55} + 6c_0D_{55} + A_{55}\right)(1+\bar{f})\right)k^2 - 2ic_0^2L_{11}\bar{f}'k^3$$

$$-c_0^2L_{11}(1+\bar{f})k^4 \ ,$$

$$m_{23} = \left(6c_0D_{55} + 9c_0^2H_{55} + A_{55}\right)\bar{f}' - \left(c_0I_4 + c_0^2I_6\right)\bar{f}'\omega^2 + i\Bigg(c_0H_{11}\bar{f}''+$$

$$c_0^2L_{11}\bar{f}'' - \left(-c_0^2I_6\omega^2 - c_0I_4\omega^2 + 6c_0D_{55} + 9c_0^2H_{55} + A_{55}\right)(1+\bar{f})\Bigg)k$$

$$+ \left(2c_0H_{11} + 2c_0^2L_{11}\right)\bar{f}'k^2 - i\left(c_0H_{11} + c_0^2L_{11}\right)(1+\bar{f})k^3 \; .$$

$$m_{31} = (I_1 + c_0I_3)(1+\bar{f})\omega^2 - i(B_{11} + c_0I_{11})\bar{f}'k - (B_{11} + c_0F_{11})(1+\bar{f})k^2 \; ,$$

$$m_{32} = -i\left(\left(c_0I_4 + c_0^2I_6\right)\omega^2k - \left(6c_0D_{55} + A_{55} + 9c_0^2H_{55}\right)\right)(1+\bar{f})k$$

$$-\left(c_0H_{11} + c_0^2L_{11}\right)\bar{f}'k^2 + i\left(c_0H_{11} + c_0^2L_{11}\right)(1+\bar{f})k^3 \; ,$$

$$m_{33} = \left(I_2 + 2c_0I_4 + c_0^2I_6\right)(1+\bar{f})\omega^2 - i\left(D_{11} + 2c_0H_{11} + c_0^2L_{11}\right)\bar{f}'k$$

$$-\left(D_{11} + 2c_0H_{11} + c_0^2L_{11}\right)(1+\bar{f})k^2 - \left(A_{55} + 6c_0D_{55} + 9c_0^2H_{55}\right)(1+\bar{f}) \; .$$

Longitudinal wave dispersion

- For analyzing purely longitudinal wave dispersion, we set $m_{11} = 0$, that is,

$$I_0(1+\bar{f})\omega^2 - iA_{11}\bar{f}'k - A_{11}(1+\bar{f})k^2 = 0$$

- By solving above quadratic equation, one gets

- $$k = -\frac{i}{2}\frac{\bar{f}'}{(1+\bar{f})} \pm \sqrt{\frac{I_0\omega^2}{A_{11}} - \left(\frac{\bar{f}'}{2(1+\bar{f})}\right)^2}$$

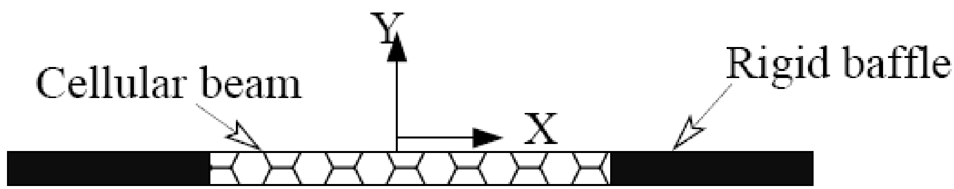
- The solution is in the form of exponentials e^{-ikx} , therefore the in-plane wave will not propagate, if Re[k]=0.
- That is possible when the descriminent is zero or negative,

$$\left|\frac{\bar{f}'}{2(\bar{f}+1)}\right| \geq \omega\sqrt{\frac{I_0}{A_{11}}}$$

By substituting expressions for $\overline{f}$ and $\overline{f'}$ one gets

$$\omega \leq \sqrt{\frac{A_{11}}{I_0}}\left|\frac{\frac{2\alpha}{\lambda}\left[\sin\left(\pi\frac{\lambda}{\lambda_m}+\theta\right)-\sin\theta\right]}{2\left[1+\left(\frac{\alpha}{\pi}\right)\left(\frac{\lambda_m}{\lambda}\right)\left\{\cos\theta-\cos\left(\theta+\pi\frac{\lambda}{\lambda_m}\right)\right\}\right]}\right|$$

- We consider a configuration, in which cellular beam is a part of a rigid baffle



Cellular laminated beam placed between two baffles

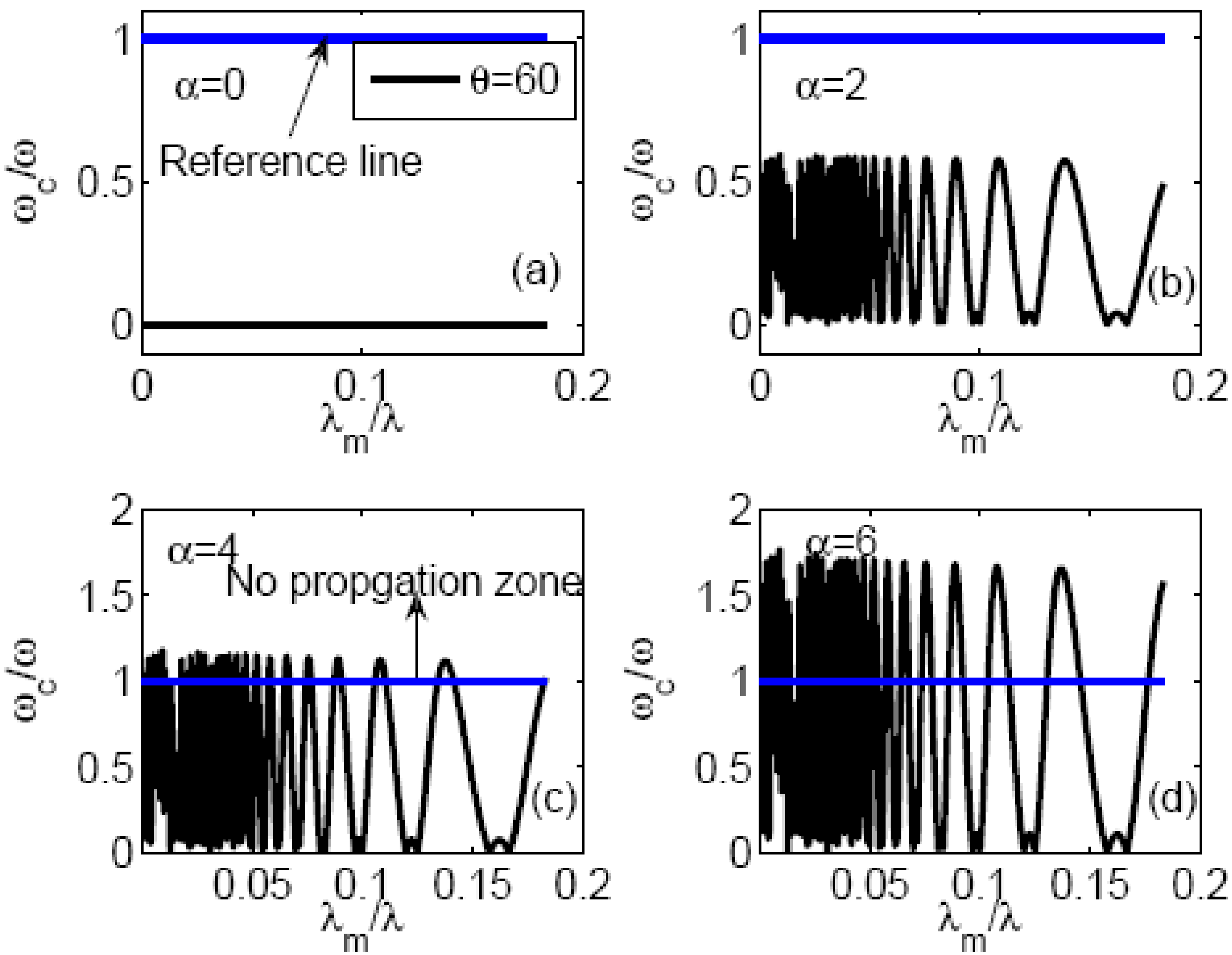
- Then the wavelength is evaluated as
- speed C= $\frac{\omega}{k}$, $\lambda = \frac{2\pi}{|k|}$ and wavenumber in the baffle k= $\sqrt{\frac{Q}{\rho}}$
- We get

$$\lambda = \frac{2\pi}{\omega}\sqrt{\frac{Q}{\rho}}$$

$$\omega_c = \sqrt{\frac{A_{11}}{I_0}}\left|\frac{\frac{2\alpha}{\lambda}\left[\sin\left(\pi\frac{\lambda}{\lambda_m}+\theta\right)-\sin\theta\right]}{2\left[1+\left(\frac{\alpha}{\pi}\right)\left(\frac{\lambda_m}{\lambda}\right)\left\{\cos\theta-\cos\left(\theta+\pi\frac{\lambda}{\lambda_m}\right)\right\}\right]}\right|$$

- When $\omega_c \geq \omega$, the inplane wave will not propagate. Therefore the frequency band over which wave will not propagate is defined as stop band, and, the contrary is the pass band.
- Use of stopbands:
- One can design a cellular beam to operate within a frequency band matching with stopband, thus one can suppress the inplane wave propagation through the cellular beam.
- An aluminum beam of 30 mm X 30 mm cross section and 60 cm length is chosen. And λ_m=8 mm is taken. Critical frequency is evaluated as a function of frequency for various stiffness modulation coefficients.

Stopband and passband analysis for inplane wave



Flexural-Shear wave dispersion

- Transverse flexural-shear wave dispersion

we get by setting,

$$\begin{bmatrix} m_{22} & m_{23} \\ m_{32} & m_{33} \end{bmatrix} \begin{Bmatrix} \tilde{w} \\ \tilde{\phi} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

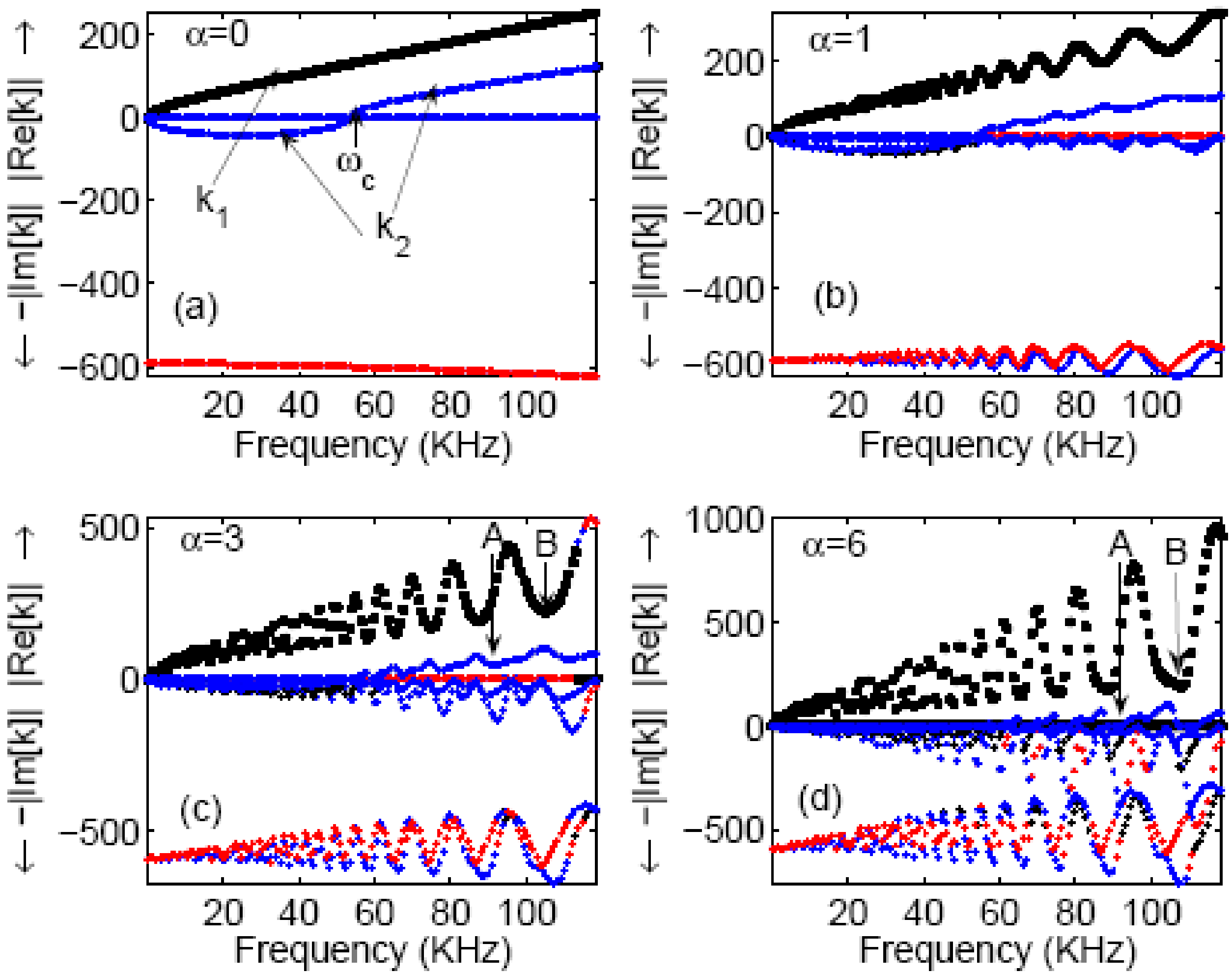
Characteristic equation,

$$m_{22}m_{33} - m_{32}m_{23} = 0 \; .$$

This can be expressed in polynomial of wavenumber k as

$$X_6k^6 + X_5k^5 + X_4k^4 + X_3k^3 + X_2k^2 + X_1k + X_0 = 0$$

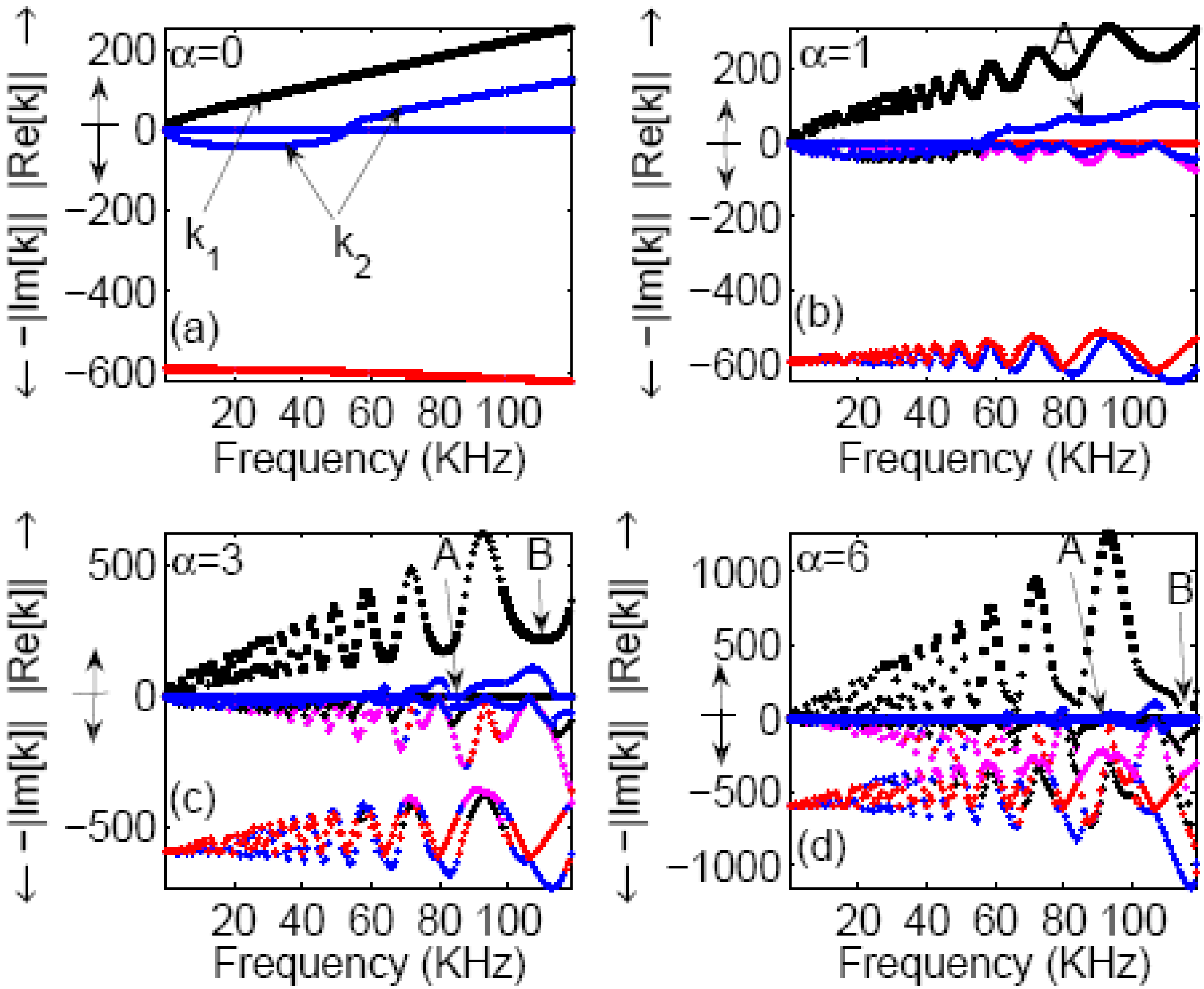
Flexural-shear wave dispersion relations for various values of α, for a flexural wave disturbance from the baffle



$$\lambda = \frac{2\pi}{\omega}\sqrt{\frac{Q}{\rho}}$$

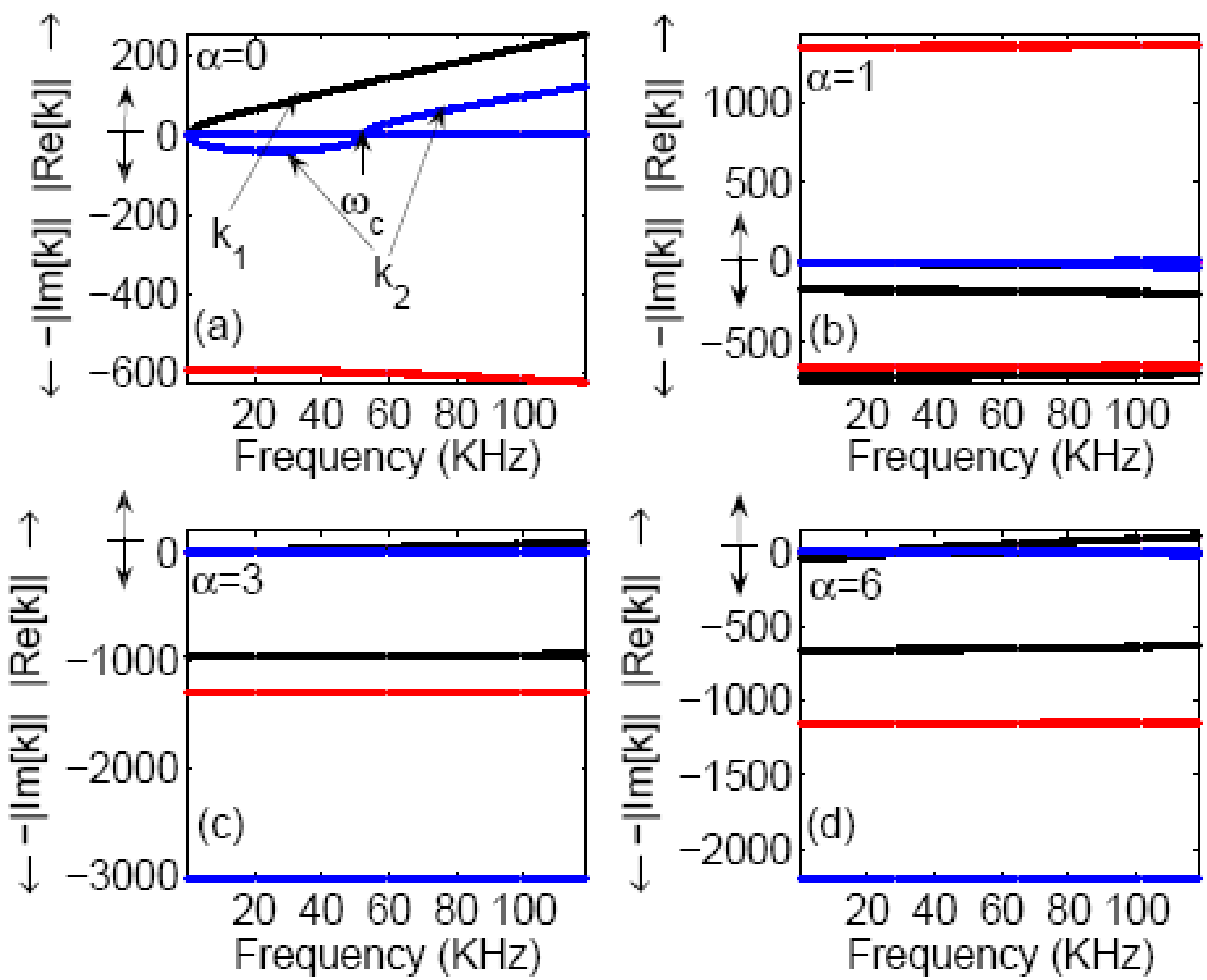
- This figure is when, $Q=Q_{11}$ in evaluating λ

Flexural-shear wave dispersion relations for various values of α, for a shear disturbance.

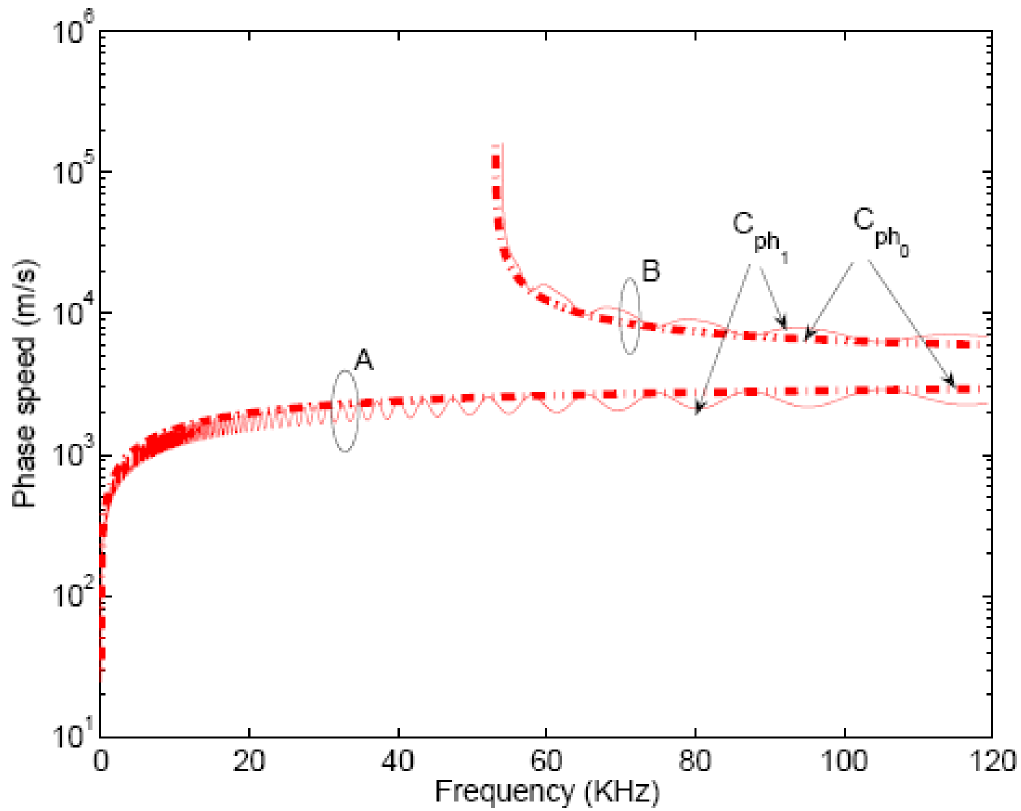


$$\lambda = \frac{2\pi}{\omega}\sqrt{\frac{Q}{\rho}}$$

Dispersion relations in the cellular beam for the case when $\lambda = \lambda_m$,



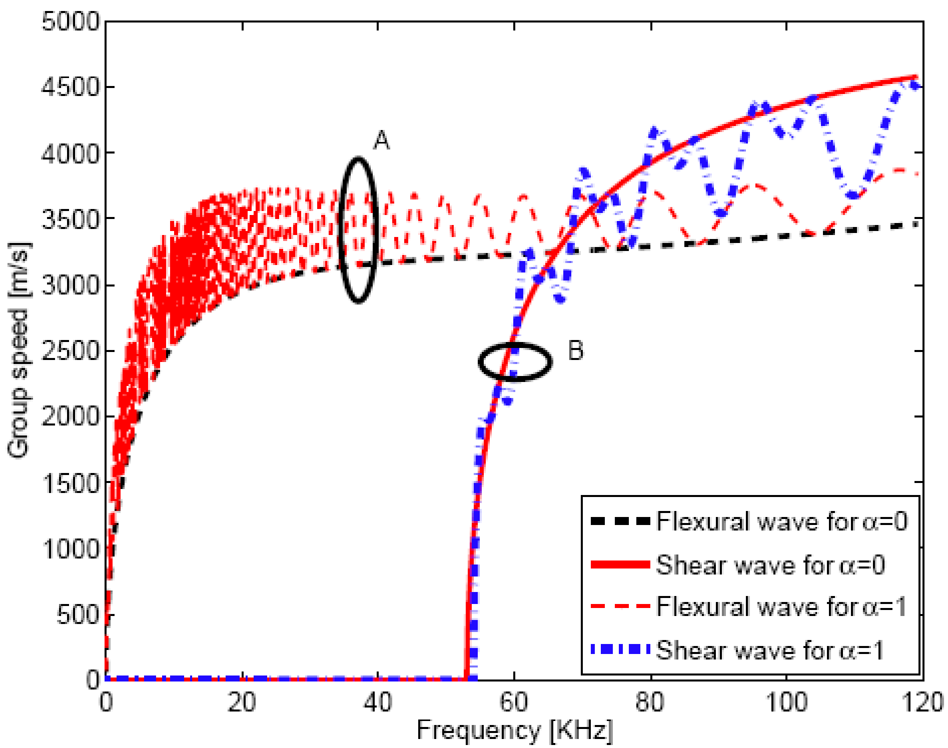
- From the dispersion relations, phase speeds are obtained
- Phase speed $C_p = \omega/k$.



Group speeds in the cellular beam

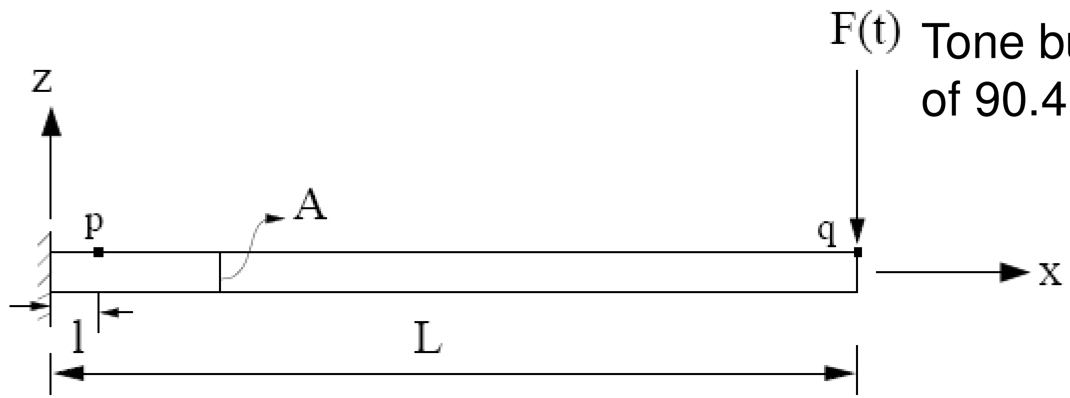
- Group speeds are obtained by differentiating characteristic equation w.r.t. ω.

$$C_g = Re\left[-\frac{6X_6k^5+5X_5k^4+4X_4k^3+3X_3k^2+2X_2k+X_1}{k^4\frac{dX_4}{d\omega}+k^3\frac{dX_3}{d\omega}+k^2\frac{dX_2}{d\omega}+k\frac{dX_1}{d\omega}+\frac{dX_0}{d\omega}}\right]$$



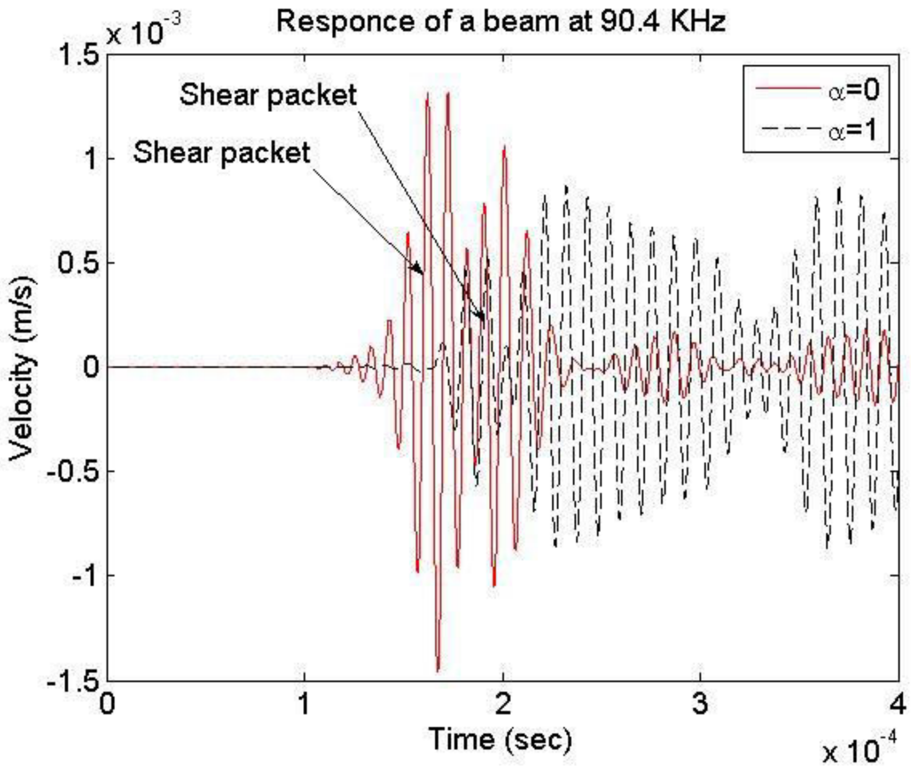
Verification of homogenization results

- This is done by using finite element simulation. A beam with same geometry and material properties is chosen as in dispersion analysis, and, modulated material properties are applied as a function of space variable ‘x’
- A modulated pulse of 90.4KHz is applied at the tip of the beam, by using an iterative solver (GMRES), the beam response is obtained at a distance of 10 cm from the root.
- Considered beam in finite element modeling:



Tone burst signal
of 90.4 KHz frequency

- The velocity response of the beam:

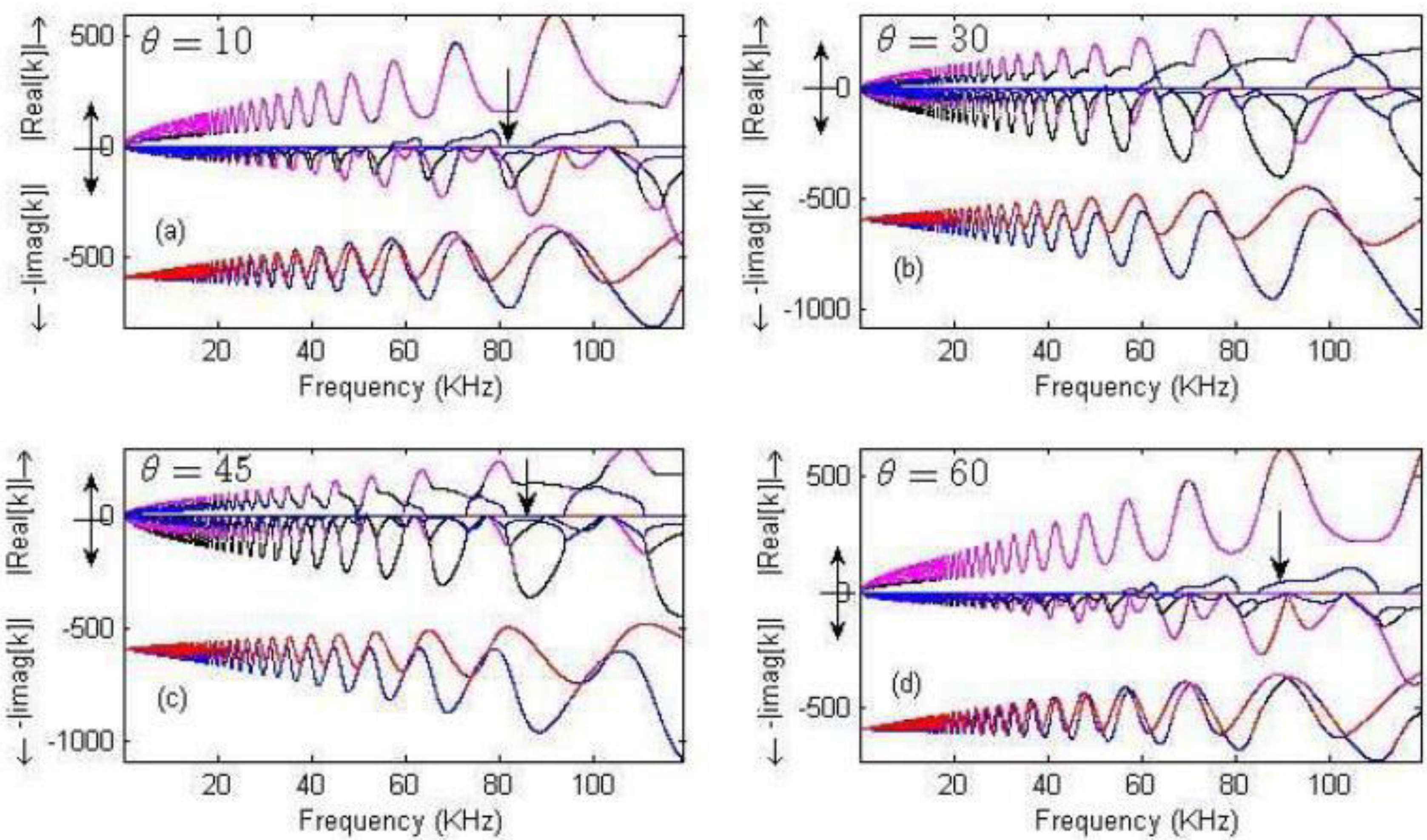


- Modulated material properties are taken, as a function of space variable x- without homogenization over wavlelength.

- Finite element simulation details:
- The geometry: Cross section 30mmx30mm, 60 cm length
- 332 triangular elements
- Time step dt=1.5x10^{-7} Sec
- Time step calculations:
- Time of travel: T=L/C;
- Where, C= $\sqrt{\dfrac{E}{\rho}}$
- $dt_{cr} \leq T/n$
- Where n is the number of time steps, (n=500)
- The analysis is done in time domain with an iterative solver (GMRES).

- Group speeds from dispersion relations at 90.4 KHz:
  - for α=0, max group speed = 4191 m/s
  - for α=1, max group speed = 3537 m/s
- Group speeds from Finite element simulation:
  - These are calculated from the travel time of tone burst signal
  - for α=0, group speed of first reached wave= 4595 m/s, this is within an error of 9.6%, which is acceptable.
  - for α=1, group speed of first reached wave= 3597 m/s, this is almost matching with the group speed estimate from dispersion analysis.
- Thus the chosen homogenization technique is verified.

- The effect of incident phase angle θ is also studied and concluded that at a phase angle of 45º, the dispersion consists of more number of stop and pass band patterns.



Concluding Remarks

- The modeling technique accounts for the wave dispersion characteristics, which is valid for the true wavelength greater than the wavelength used for homogenization.

- Corresponding to the stop bands (Re[k] tending to zero), one can estimate the modulation parameters and then solve an optimization problem to arrive at the physical details of the cell structure (e.g., honeycomb with filling patterns).

- A more accurate method would be to take into account the cell wall kinematics, cell cavity pressure oscillation and homogenize the cell constitutive response first and then carry out the proposed homogenization scheme in the suitable wavelength scale. This is a research under progress.